

Hybrid Sampling Algorithms for Dynamic Graph Sampling

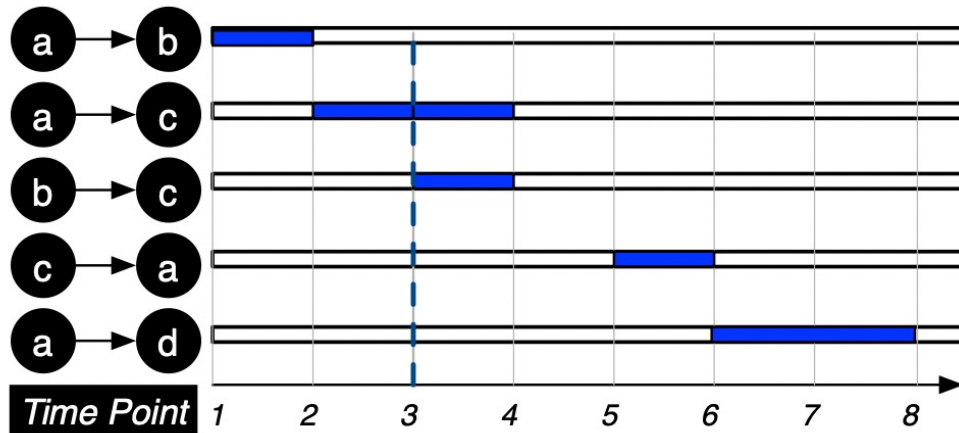
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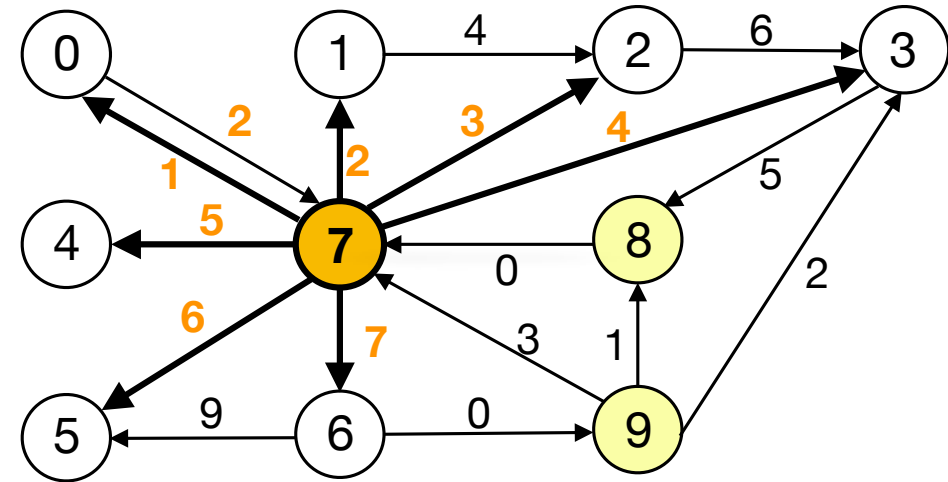
Background

- When do we need to use the dynamic graph sampling?
- Temporal Graph Random Walk
 - Dynamic graph sampling caused by time limit
- Evolving Graph Random Walk
 - Dynamic graph sampling caused dynamic edge modification

What's temporal graphs ?



Flight map



Social network

Temporal graph: Graph with time instances

Temporal path: Time increasing on paths

Random walks on temporal graphs

Random walk: A temporal path starting from a certain edge by randomly sampling

Candidate edge set: candidate edges for each sampling (satisfy time constraints)

Decided by previous arriving time

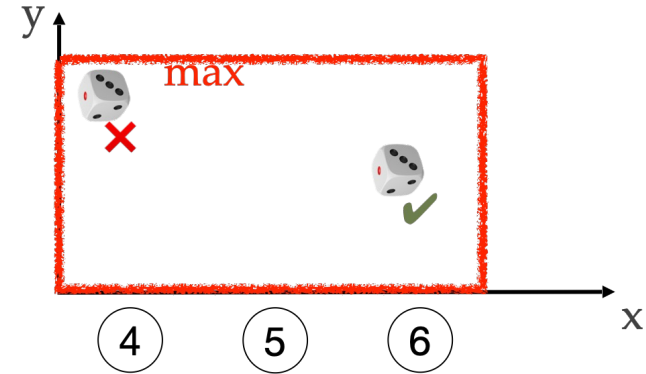
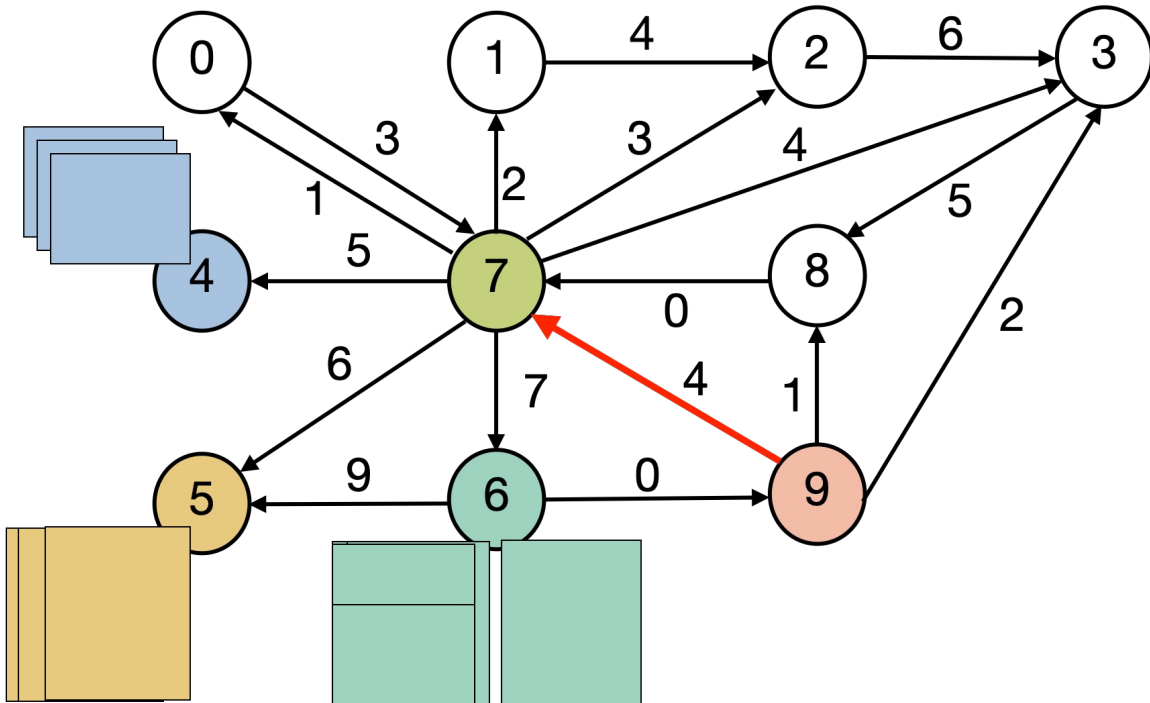
$$\Gamma_t(u) = \{(u, v_i, t_i) \mid (u, v_i, t_i) \in N(u), t_i > t\}$$

Transition probability: probability distribution of edges on each candidate edge set

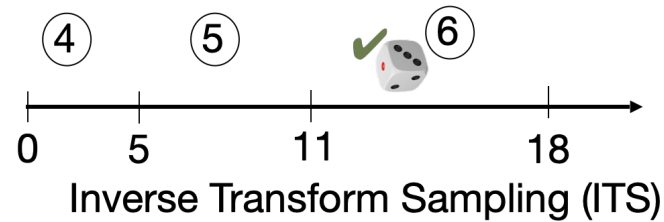
Decide by previous arriving time

$$P((u, v_i, t_i)) = \beta_{(u, v_i)} \cdot \frac{\delta((u, v_i, t_i))}{\sum_{(u, v_j, t_j) \in \Gamma_t(u)} \delta((u, v_j, t_j))}$$

Question: Random Walk $9 \rightarrow 7 \rightarrow ?$



Rejection method



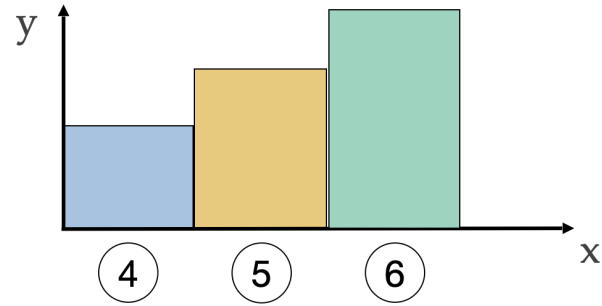
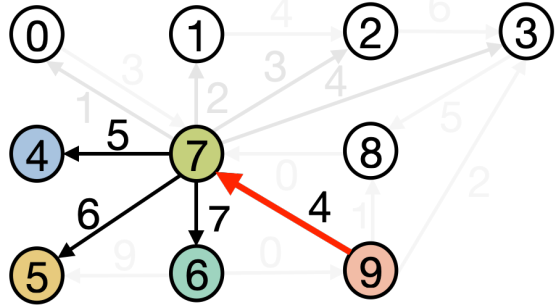
Inverse Transform Sampling (ITS)



Alias method

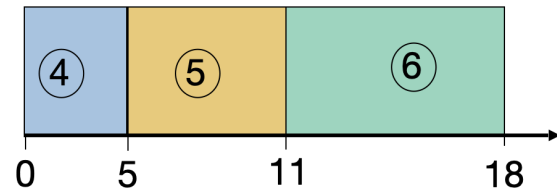
[1] Huan, C and Pandey, S. and Liu, H etc. 2023. TEA: A General Purpose Temporal Graph Random Walk Engine. Eurosys '23

Challenges

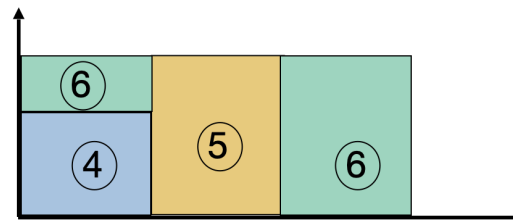


Rejection method

$$\text{Bias} = \frac{e^{t_k}}{\sum e^{t_i}}$$



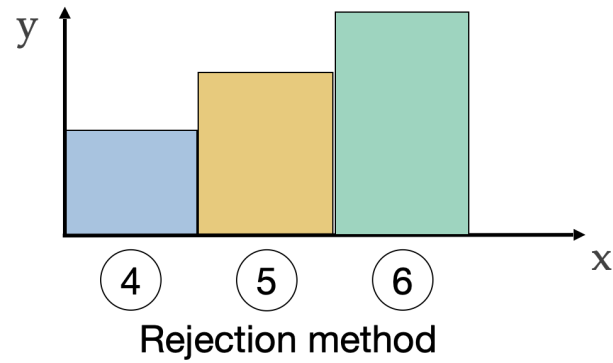
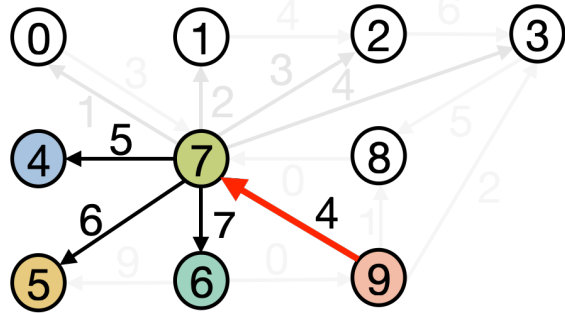
Inverse Transform Sampling (ITS)



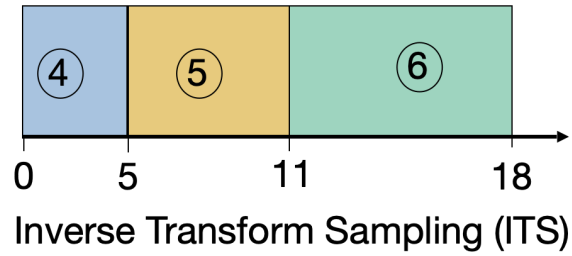
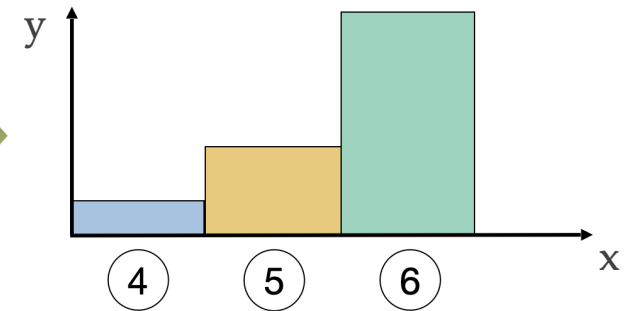
Alias method

x

Challenges

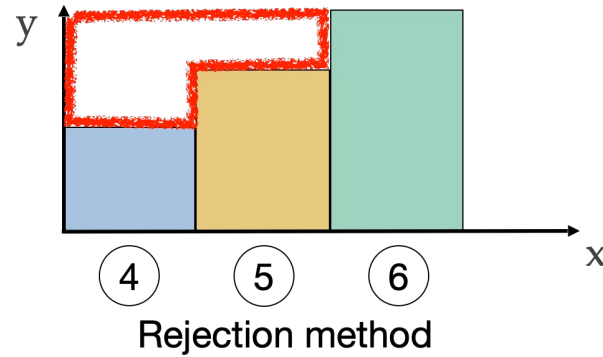
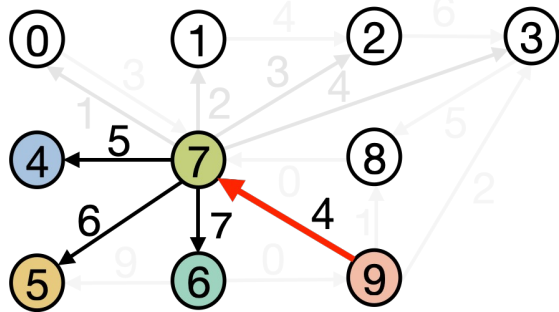


$$\text{Bias} = \frac{e^{t_k}}{\sum e^{t_i}}$$

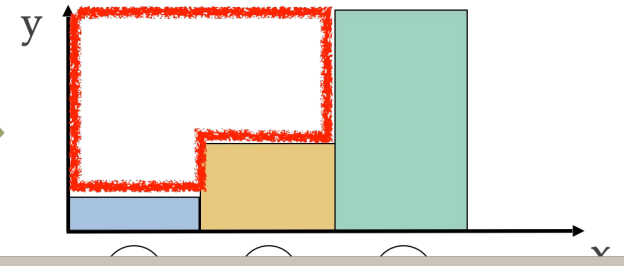


x

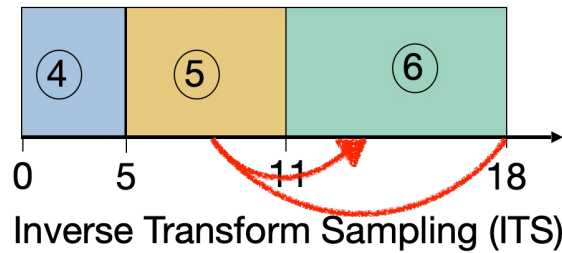
Challenges



$$\text{Bias} = \frac{e^{t_k}}{\sum e^{t_i}}$$



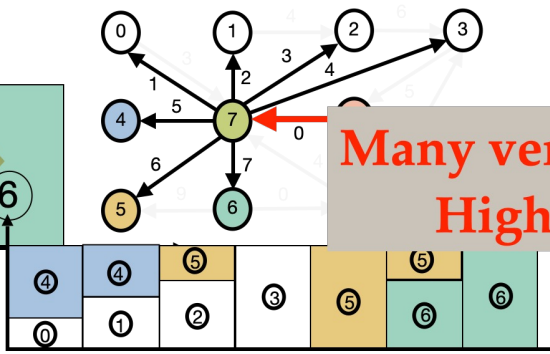
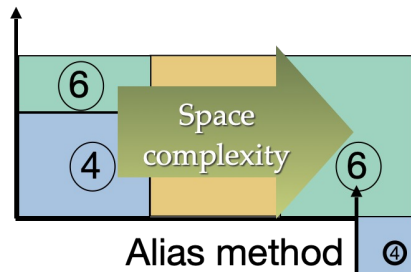
Very high rejection ratio!!!



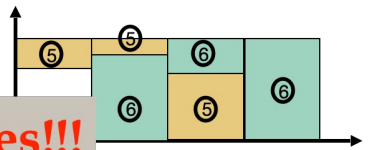
Rand=13

$$\text{Time complexity}$$

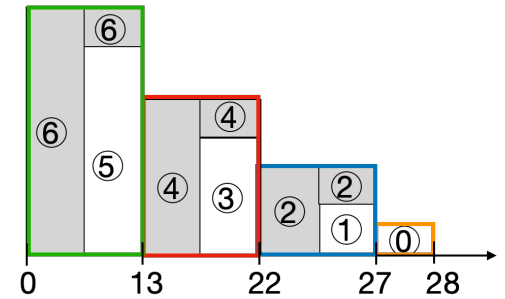
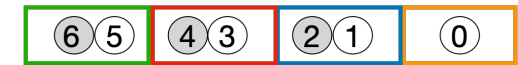
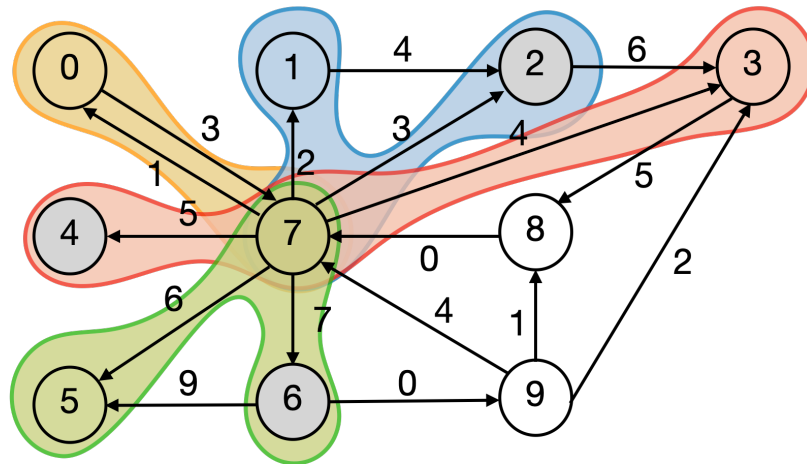
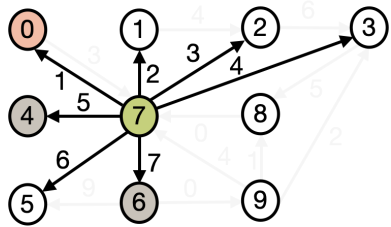
ALWAYS LogN time complexity!



**Many versions of alias tables!!!
High space complexity**

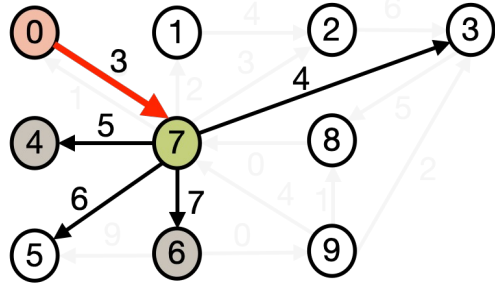


Persistent Alias Table [Eurosys '23]



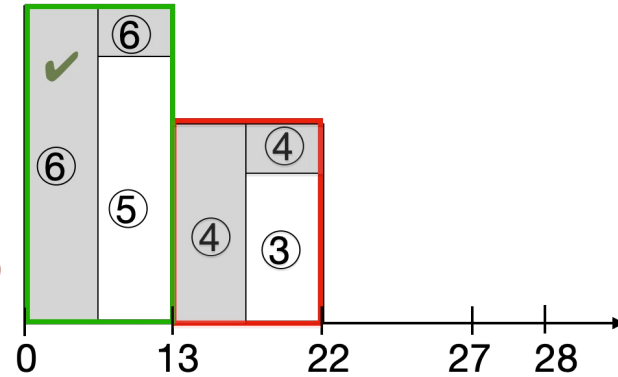
x

Sampling on Persistent Alias Table



Complete alias tables

$O(\log(N/|\text{AliasTable}|))$

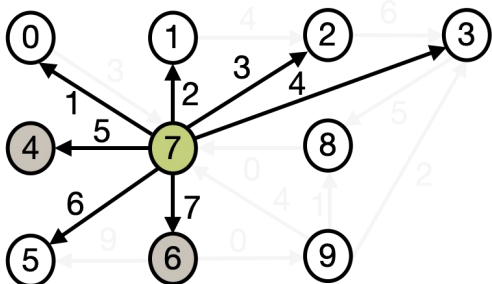


Step 1: Using ITS to select the alias table of interest.

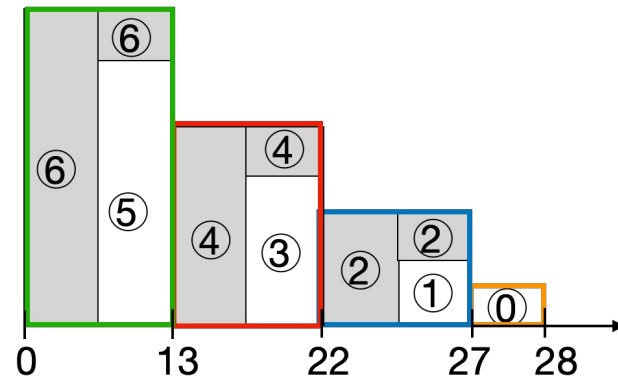
Random = 12

Step 2. Using alias method to select the vertex.

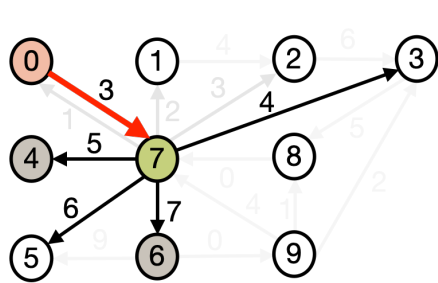
Vertex = ⑥



Incomplete alias tables

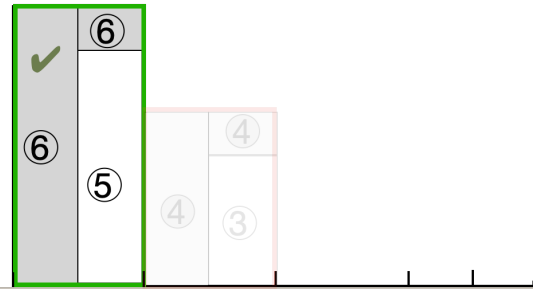


Sampling on Persistent Alias Table



Complete alias tables

$$O(\log(N/|\text{AliasTable}|))$$

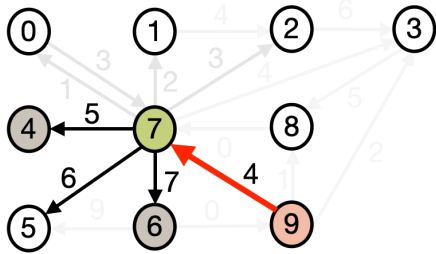


Step 1: Using ITS to select the alias table of interest.

Random = 12

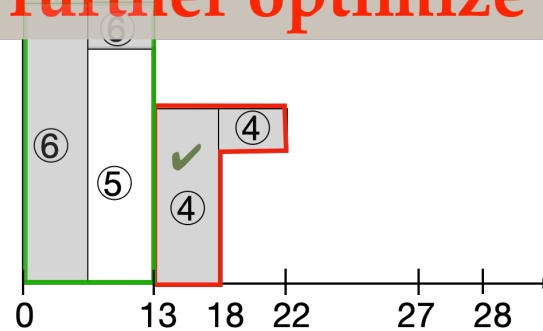
Step 2. Using alias method to select the vertex.

Since $|\text{AliasTable}|$ affects the complexity,
We further optimize $|\text{AliasTable}|!$



Incomplete alias tables

$$O(\log(N/|\text{AliasTable}|) + \log|\text{AliasTable}|)$$



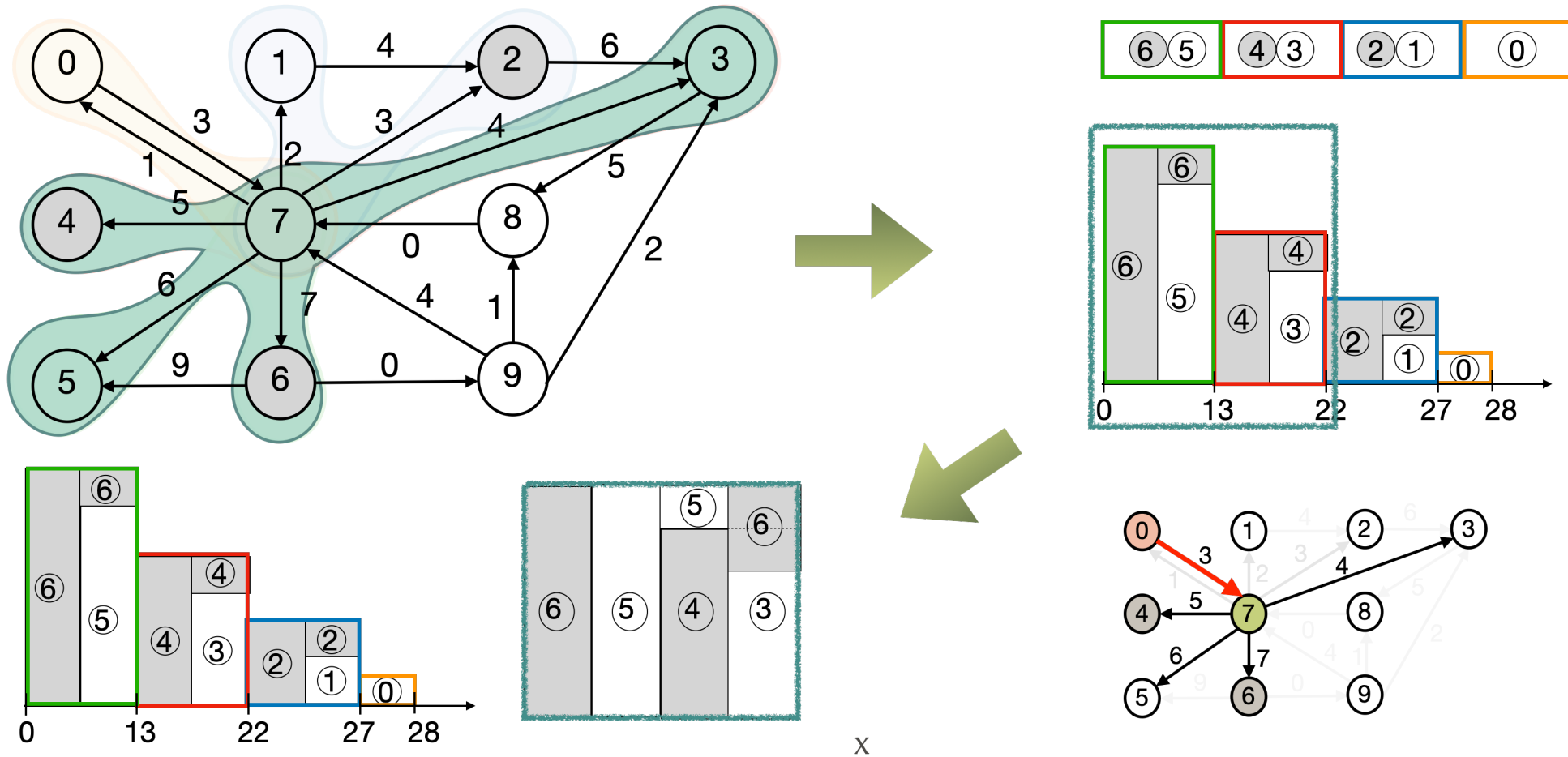
Step 1: Using ITS to select the alias table of interest.

Random = 14

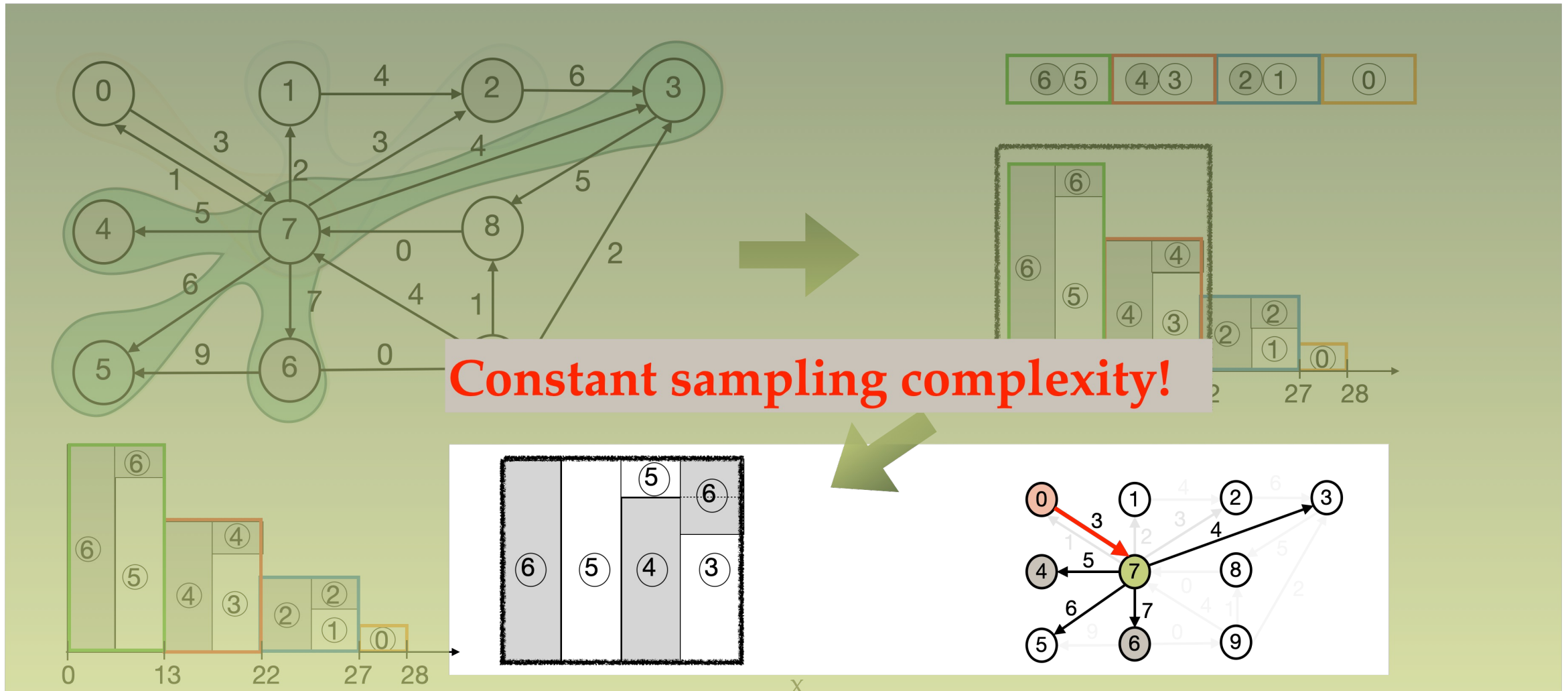
Step 2. Using ITS method to select the vertex from incomplete alias table.

Vertex = ④

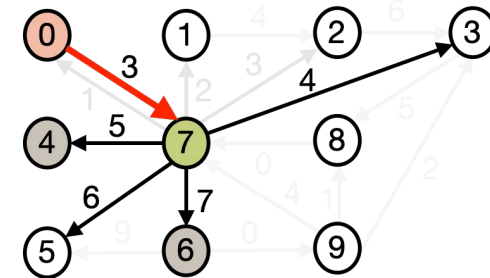
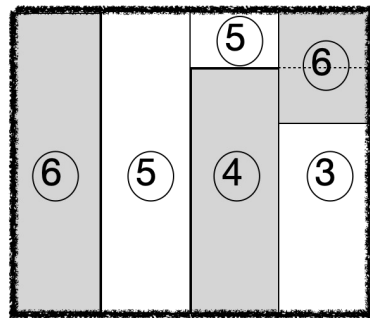
Hierarchical Persistent Alias Table



Hierarchical Persistent Alias Table



Constant sampling complexity!



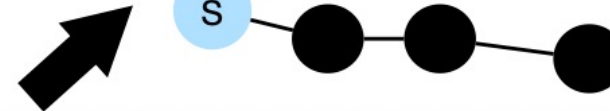
How to rapidly identify the candidate alias tables?

System overview:

Incoming random walk source node **s**



Output random walk

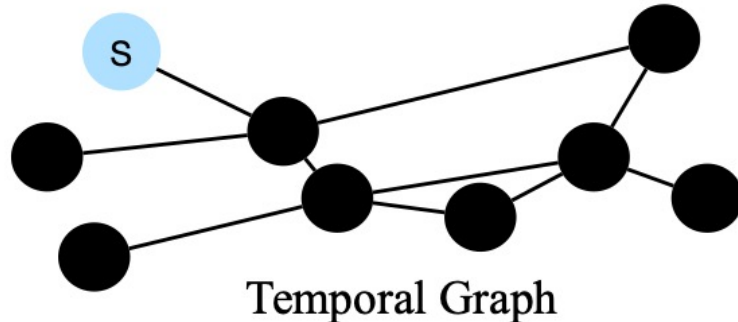


1. Access auxiliary index of node *s*;
2. Access hierarchical persistent alias table with auxiliary index;
3. Perform sampling on the hierarchical persistent alias table;
4. Using the newly selected vertex, repeat from 1;

TEA runtime sampling engine

Steps 1 - 4 continue until our random walk meets the required length.

TEA preprocessed data structures



Persistent alias table (preprocessed)

Hierarchical persistent alias table
(B+ tree, where each node is an alias table, preprocessed)

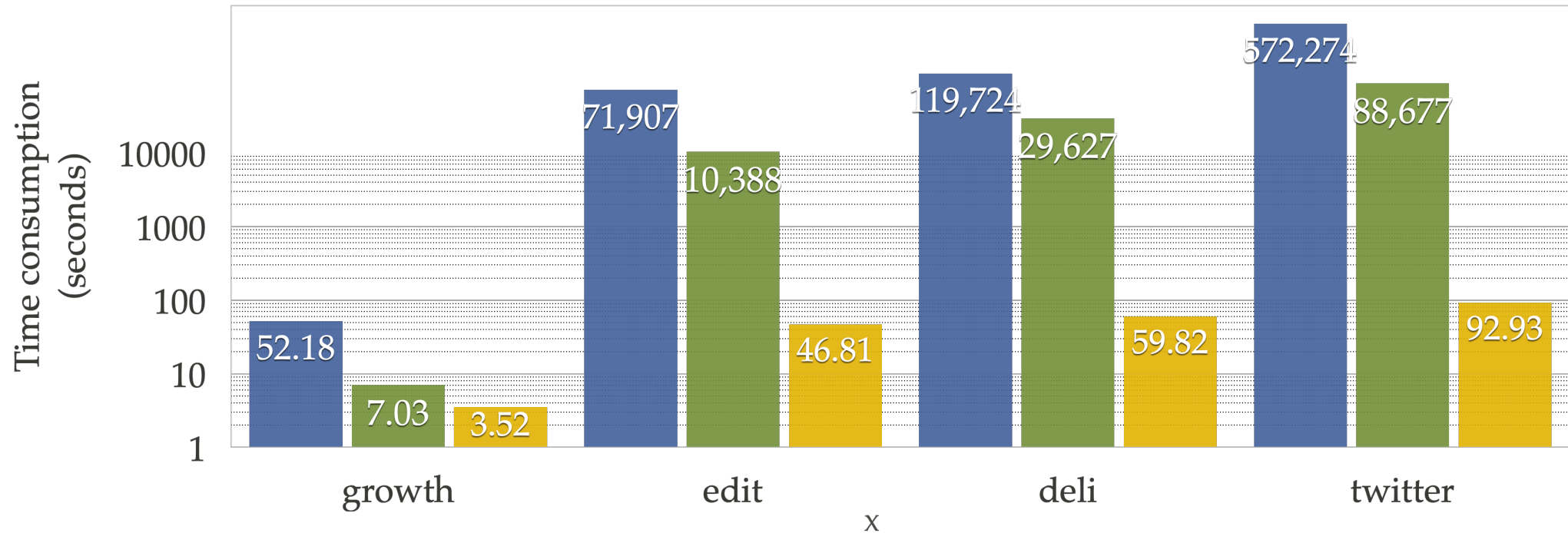
Auxiliary index
(<key value> pairs, preprocessed)

Streaming graph support

Tea vs. the State-of-the-art

- GraphWalker [ATC '20]
- KnightKing [SOSP '19]
- Our Tea

Tea enjoys higher speedups on bigger graphs!

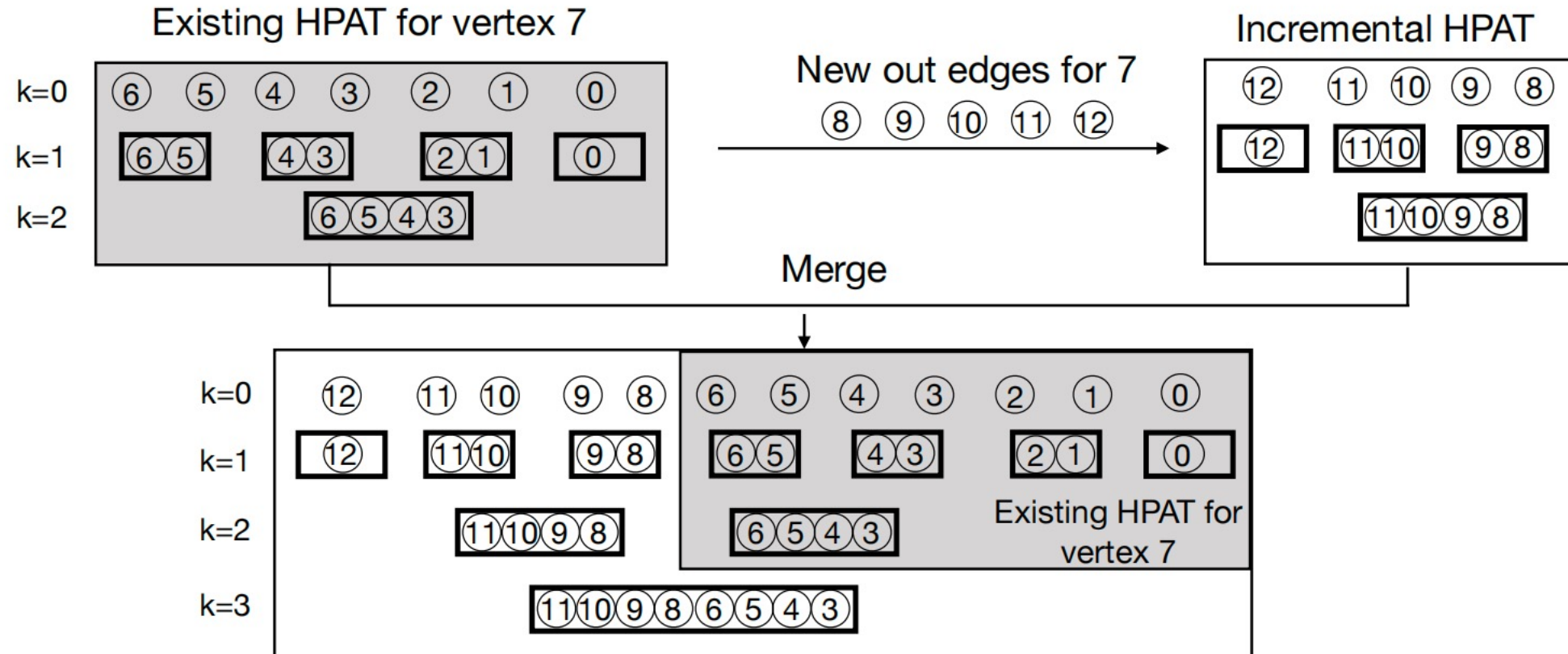


How to deal with dynamic edge modification?

Incremental HPAT Updating

- Edge Insertion
 - Create new HPATs and merge the new HPATs into old HPATs
- Edge Deletion
 - Delete the corresponding HPATs

Edge Insertion

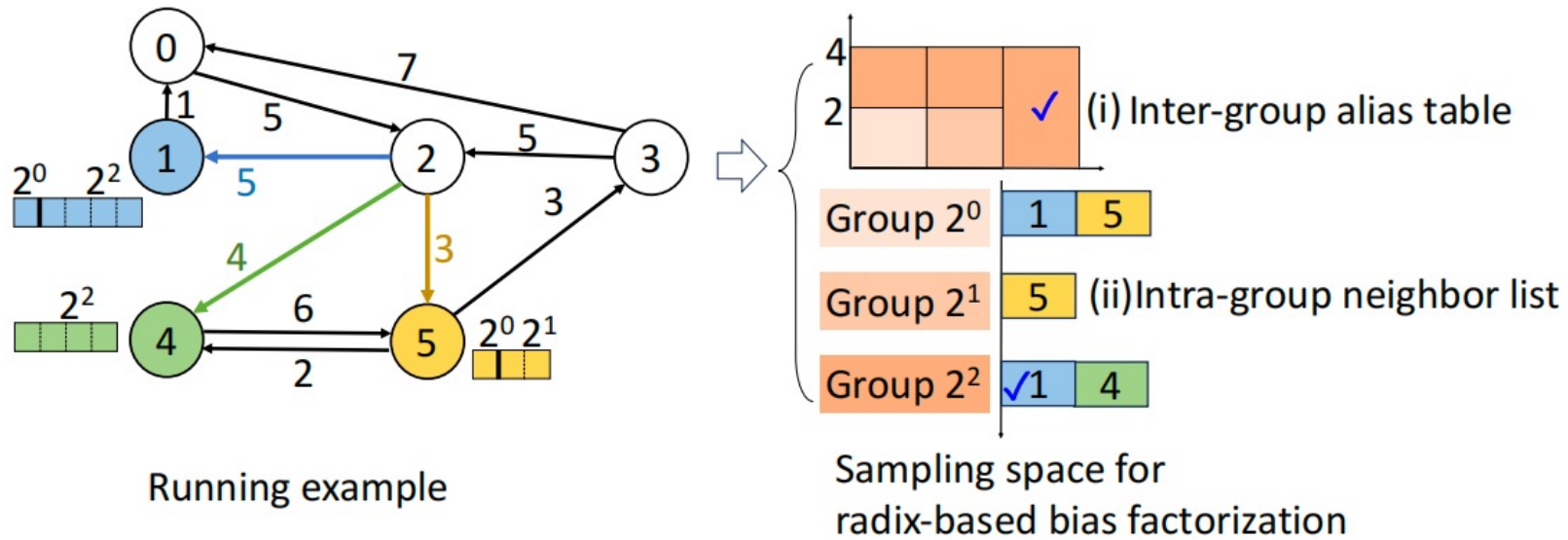


Edge Deletion

- Remove the trunks containing the deleted edge
- For example:
 - when deleting the edge from vertex 7 to $\{0,1,2,3\}$
 - the trunks containing $\{0,1,2,3\}$ will be directly deleted
 - this is correct because we arrange the edges by time decreasing and each candidate edge set length can be binary decomposition
- The time complexity of each edge deletion is $O(1)$

How to deal with evolving graph without time instances?

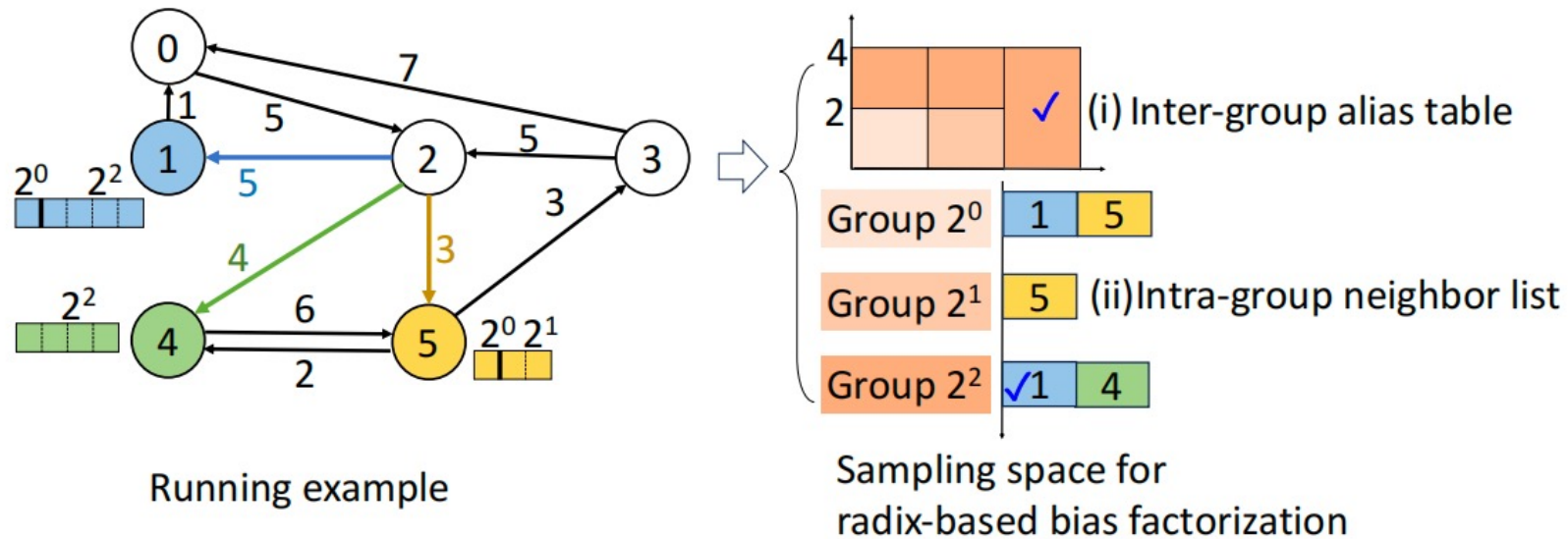
Radix-based Bias Factorization



Binary decomposition
for edge weight

$O(\log(\log(D)))$ sampling complexity

Edge insertion



$O(\log(D))$ time complexity

Thank you ~ ~