Path Representations

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University of Bayreuth

LDBC Meeting
@
SIGMOD/PODS'22

Work in progress (paper almost submission ready) with
Matthias Niewerth, Tina Popp, Stijn Vansummeren, Domagoj Vrgoč, Matthias Hofer

Some Challenges in Graph Queries

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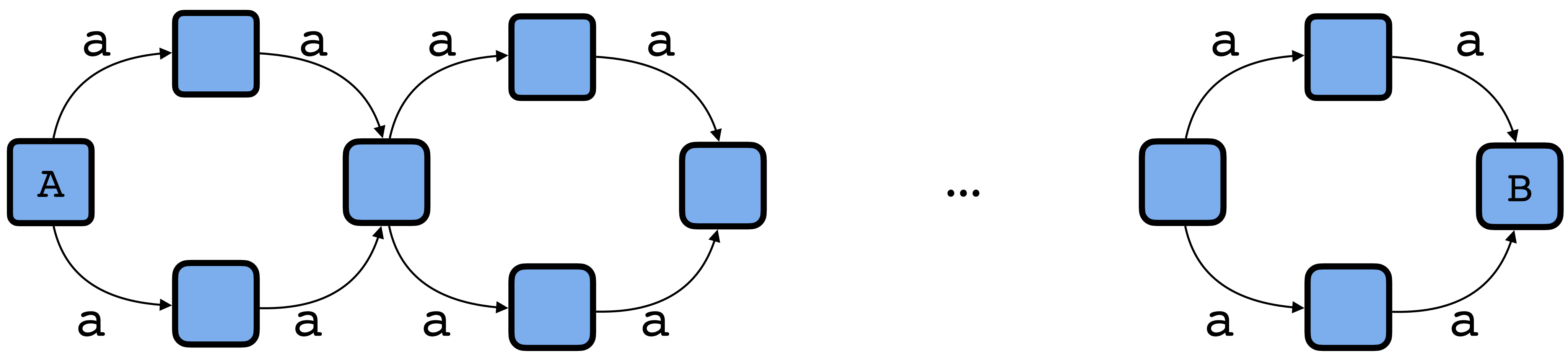

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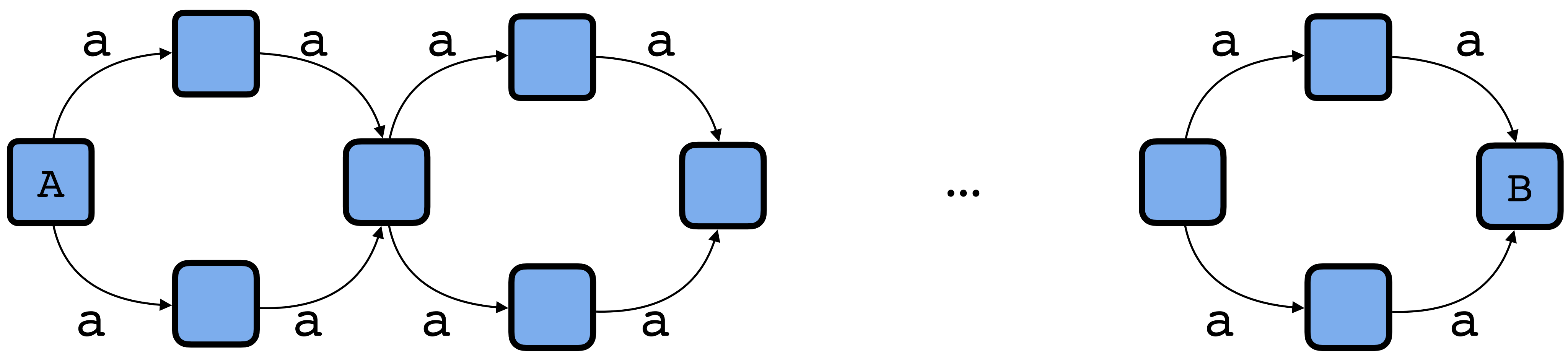

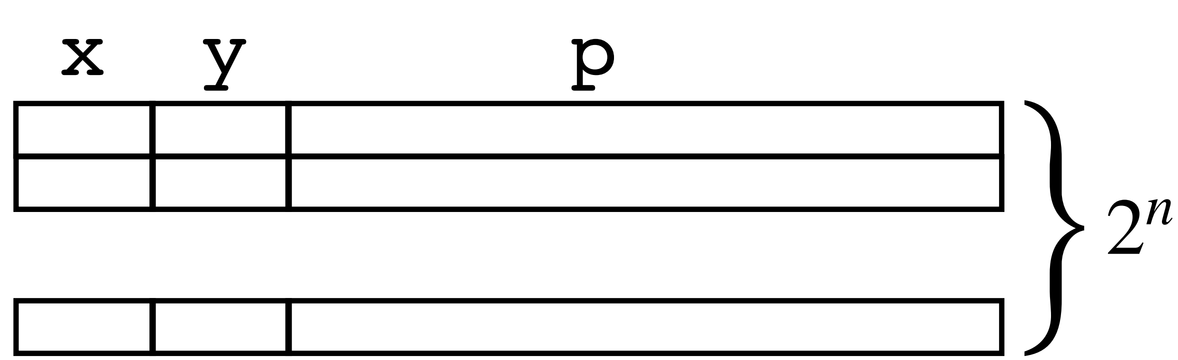

Returns 2^n many paths on a graph with $O(n)$ nodes and edges

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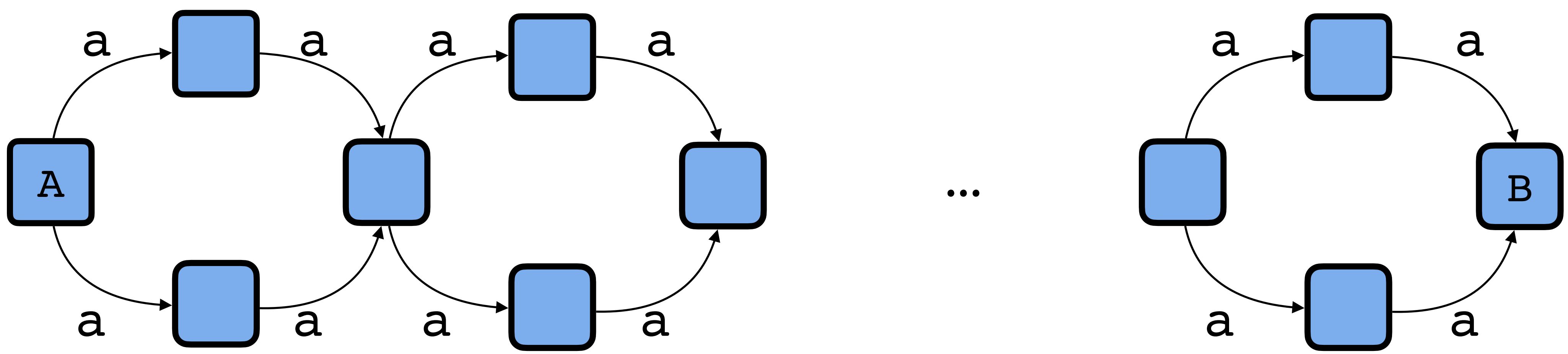

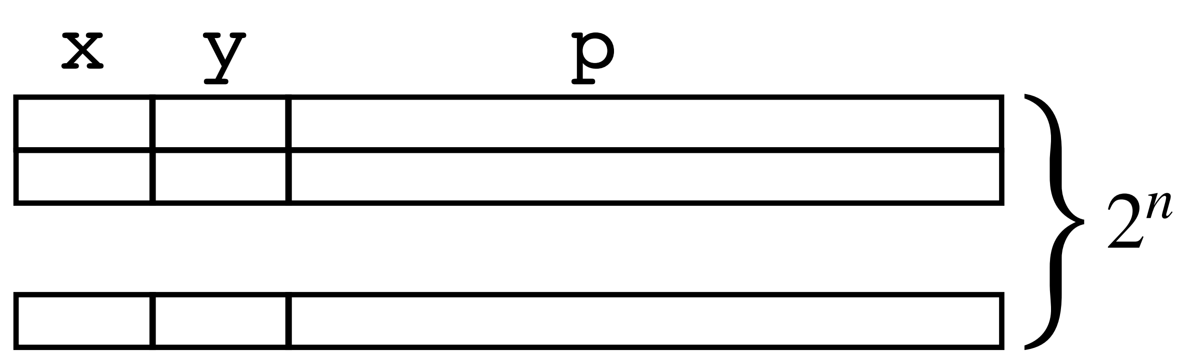

Returns 2^n many paths on a graph with $O(n)$ nodes and edges

(This is a lot more than the endpoint pairs from SPARQL and academic research)

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$$\text{Graph} \xrightarrow{\text{query}} \text{Table}$$

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vs

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Challenge exists on two levels

- representing the output of entire queries
- representing intermediate results in query plans

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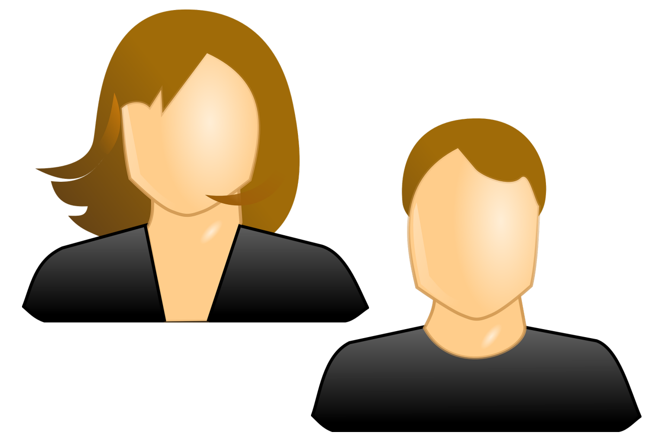

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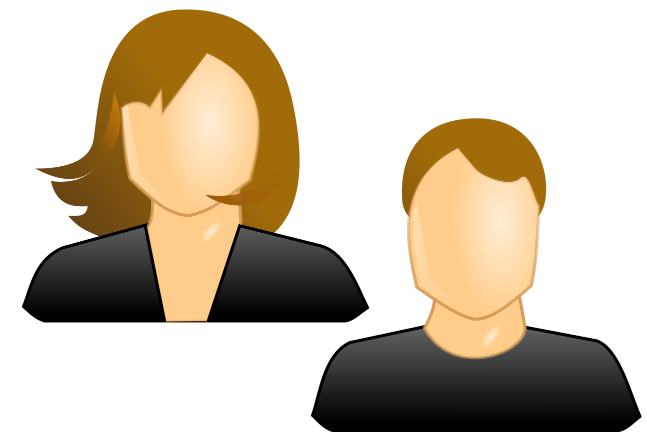

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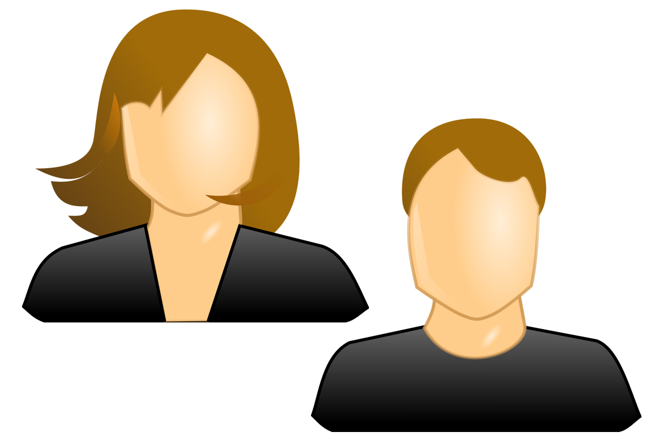

- table can be extremely large and complex
- graph projections look nice but are not lossless

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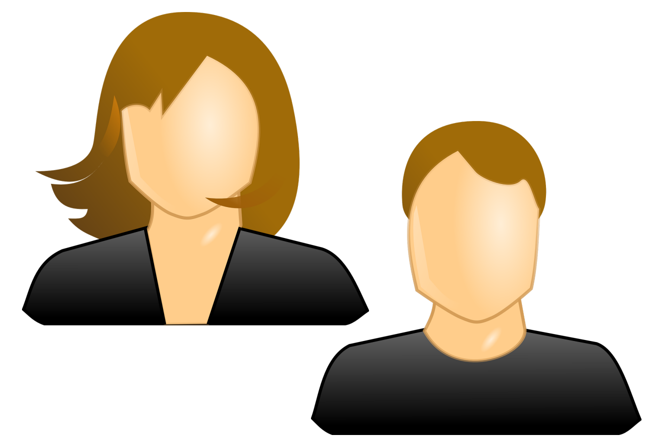

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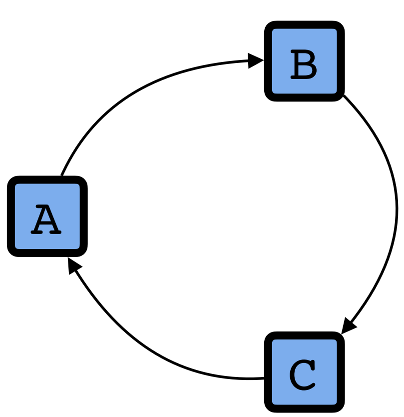

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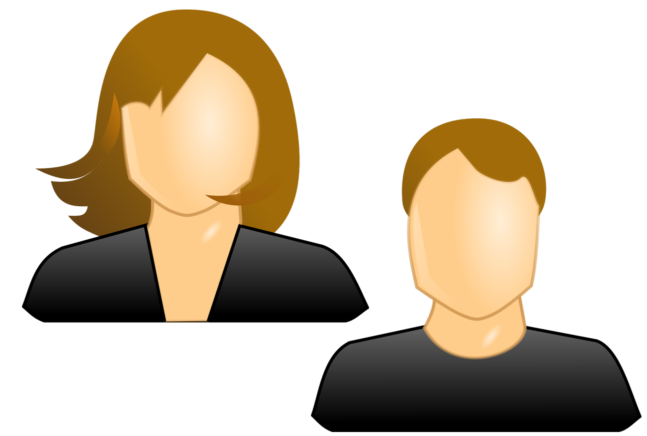

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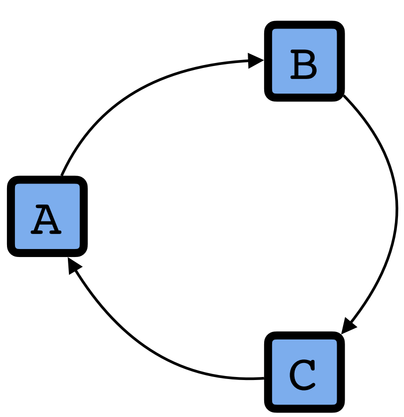

shortest
even length
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from A
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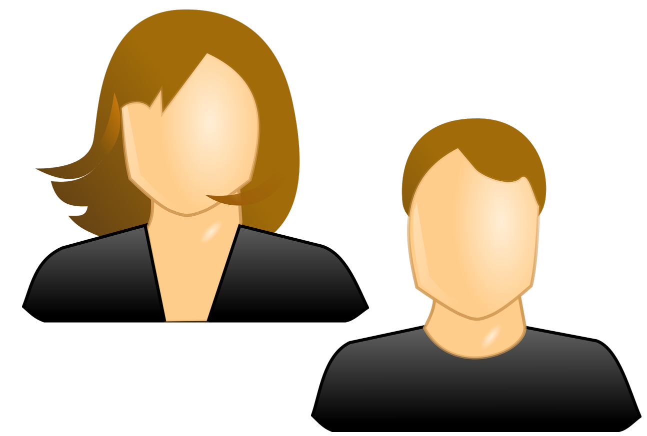

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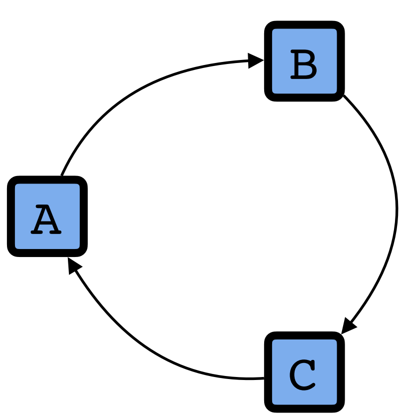

shortest even length path from A to A $\longrightarrow$ Table

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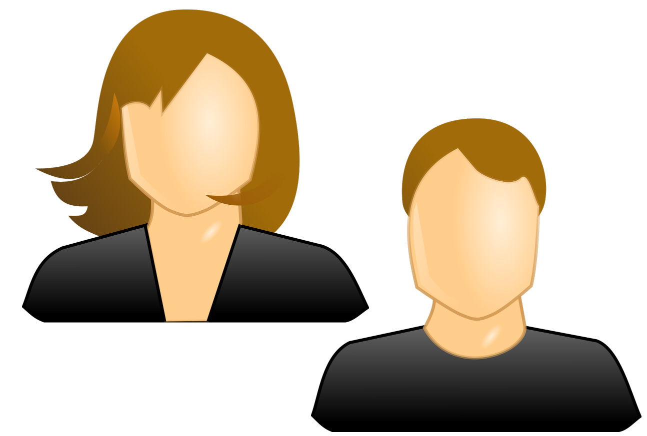

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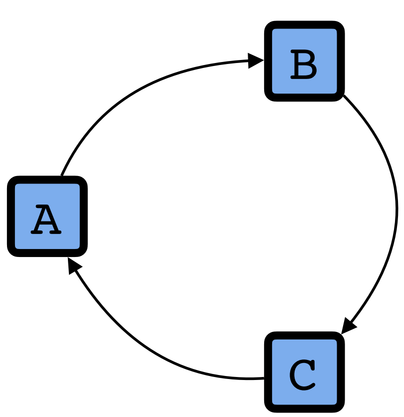

$\xrightarrow{\text{shortest even length path from A to A}}$ Table $\xrightarrow{\text{graph projection}}$

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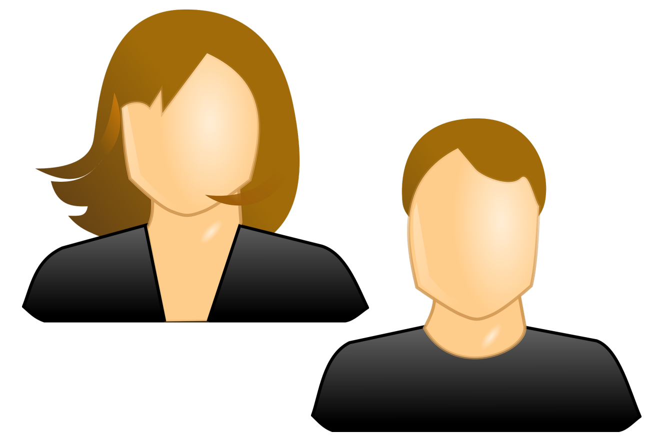

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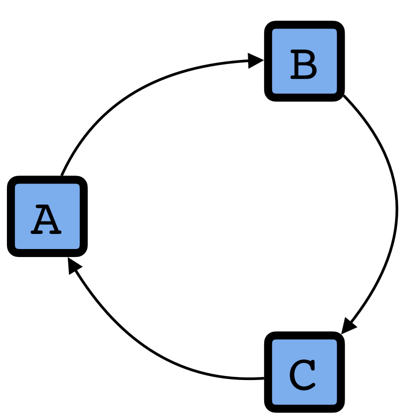

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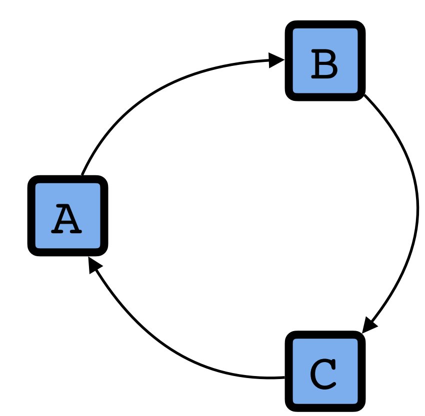
graph projection



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Present an idea that may help here

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Present an idea that may help here

- Focus on 1. and 2.
- We've done a lot of thinking but it's still work in progress
  - First paper is close to ready
- I think it's very promising
  - We'll definitely keep working on it

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Store intermediate results of queries as graphs

- Can be exponentially more succinct than the table
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- Without losing information (as opposed to graph projection)

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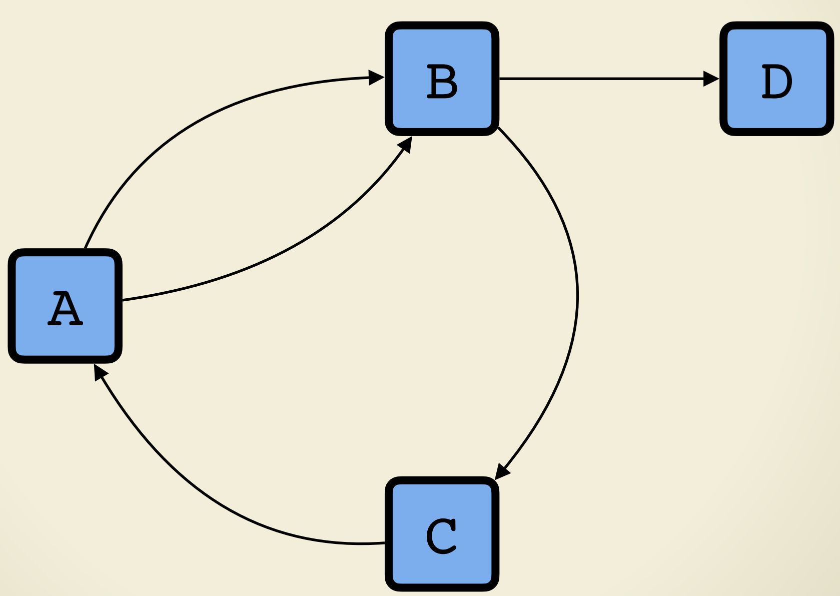

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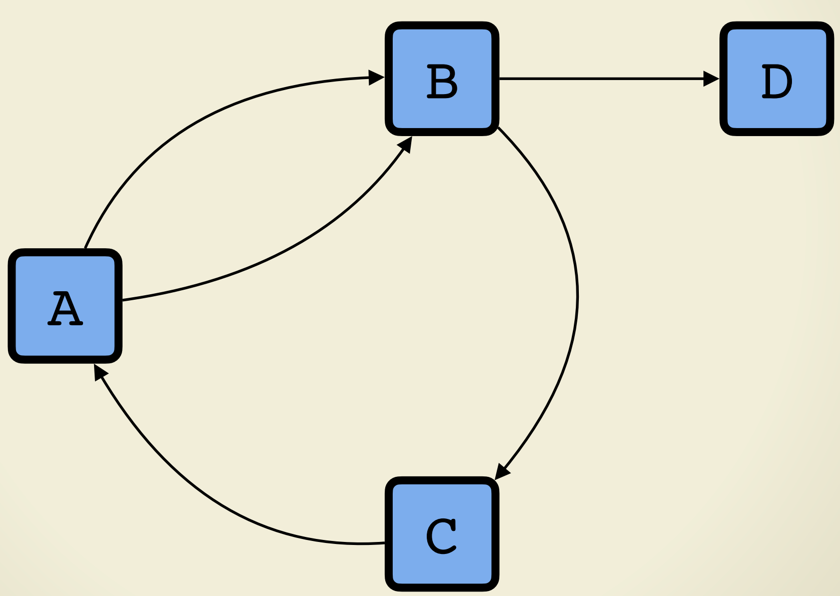
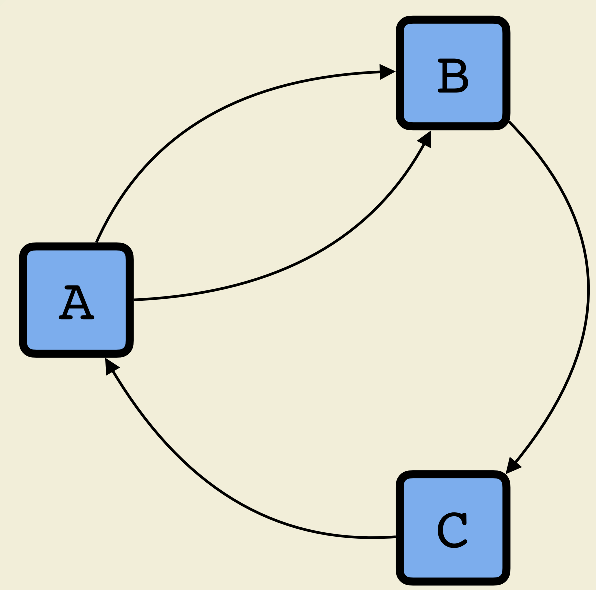


Representation of Paths in Output



"All paths from A to B in this graph"

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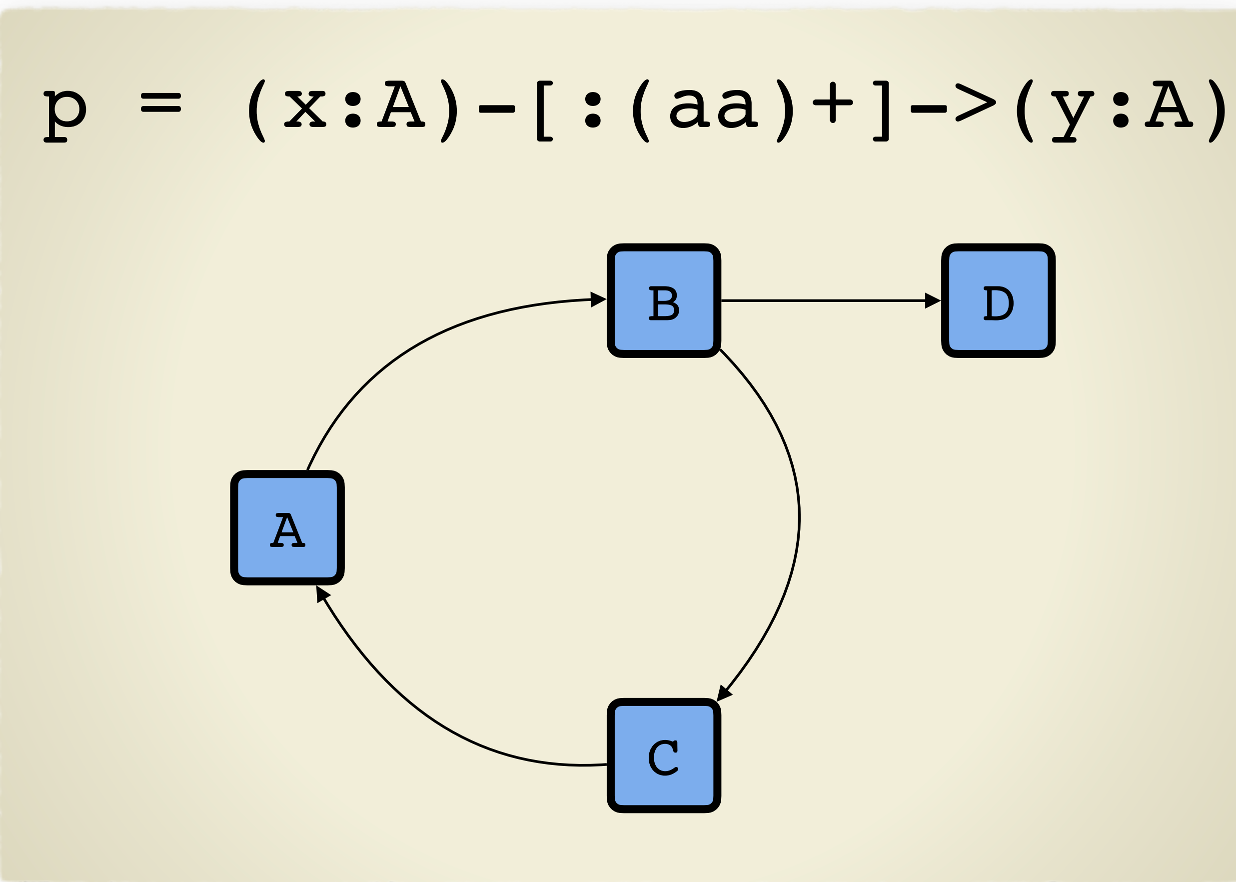

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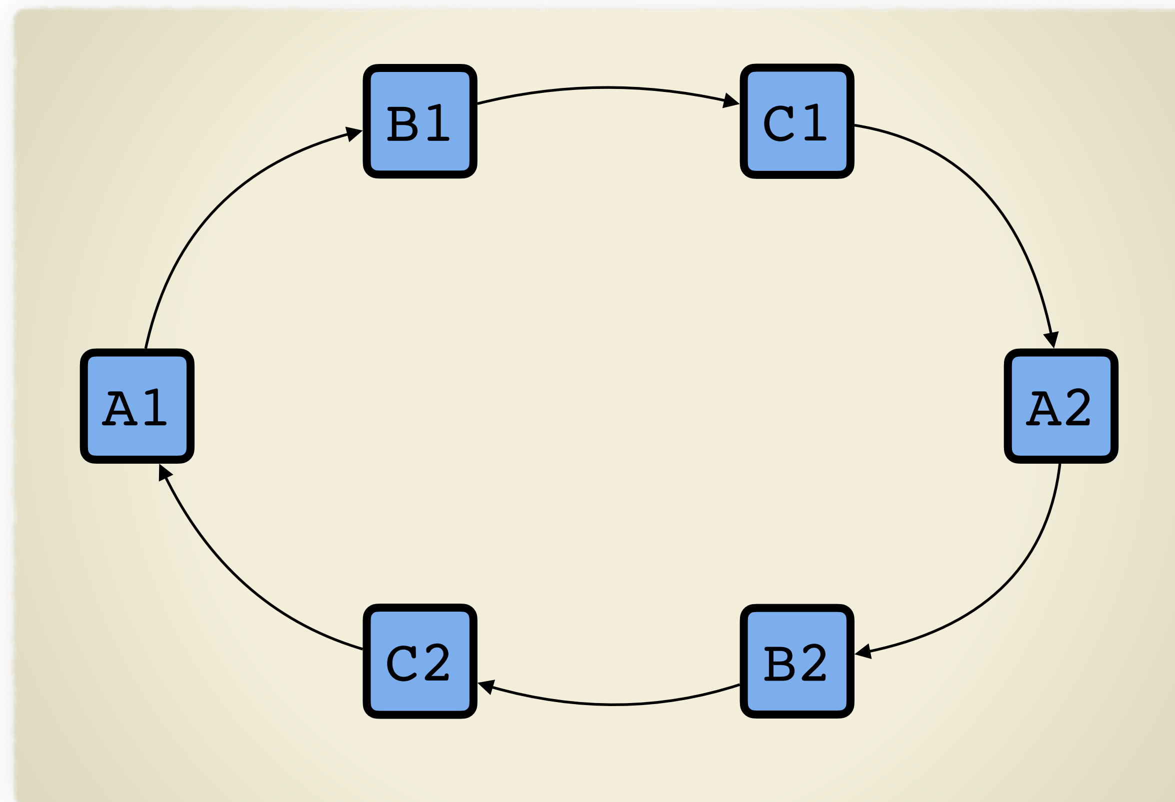

"All paths from A1 to A1 in this graph"

Path Representations

Let $G = (N_G, E_G, \eta, \lambda)$ be a graph, where

- $\eta : E_G \to (N_G \times N_G)$ maps edge ids to pairs of node ids
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Definition

A path representation over graph G is a tuple

$$R = (N, E, \eta, \gamma, S, T),$$

where

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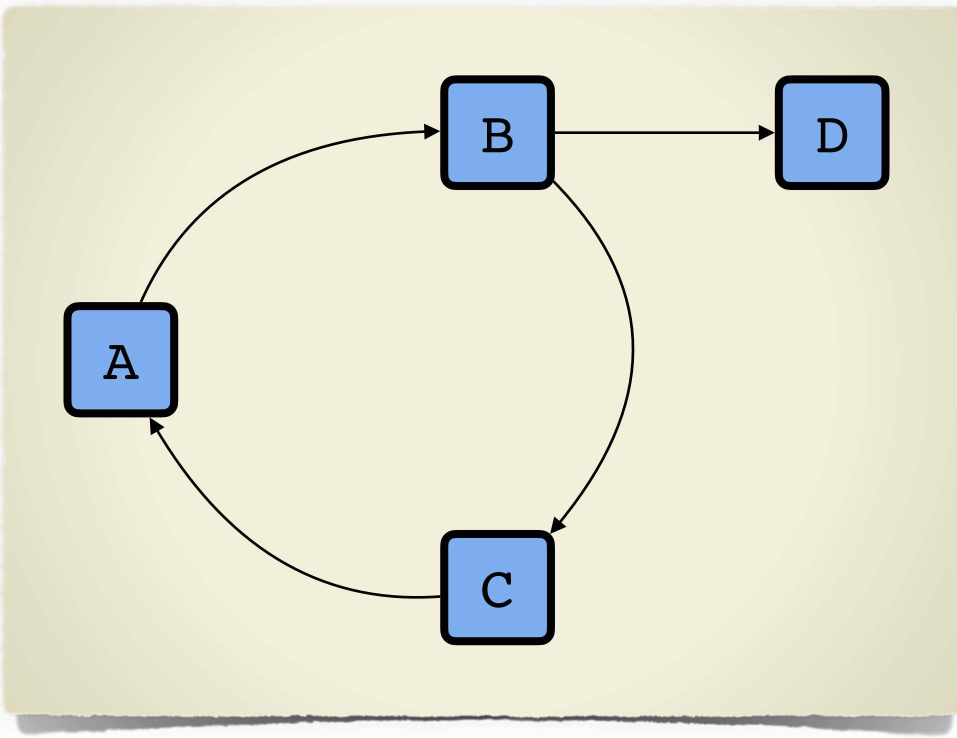
This is a lossless representation
of a set or multiset of paths in G

Path Representations: Examples

The set of even length paths from A to B in



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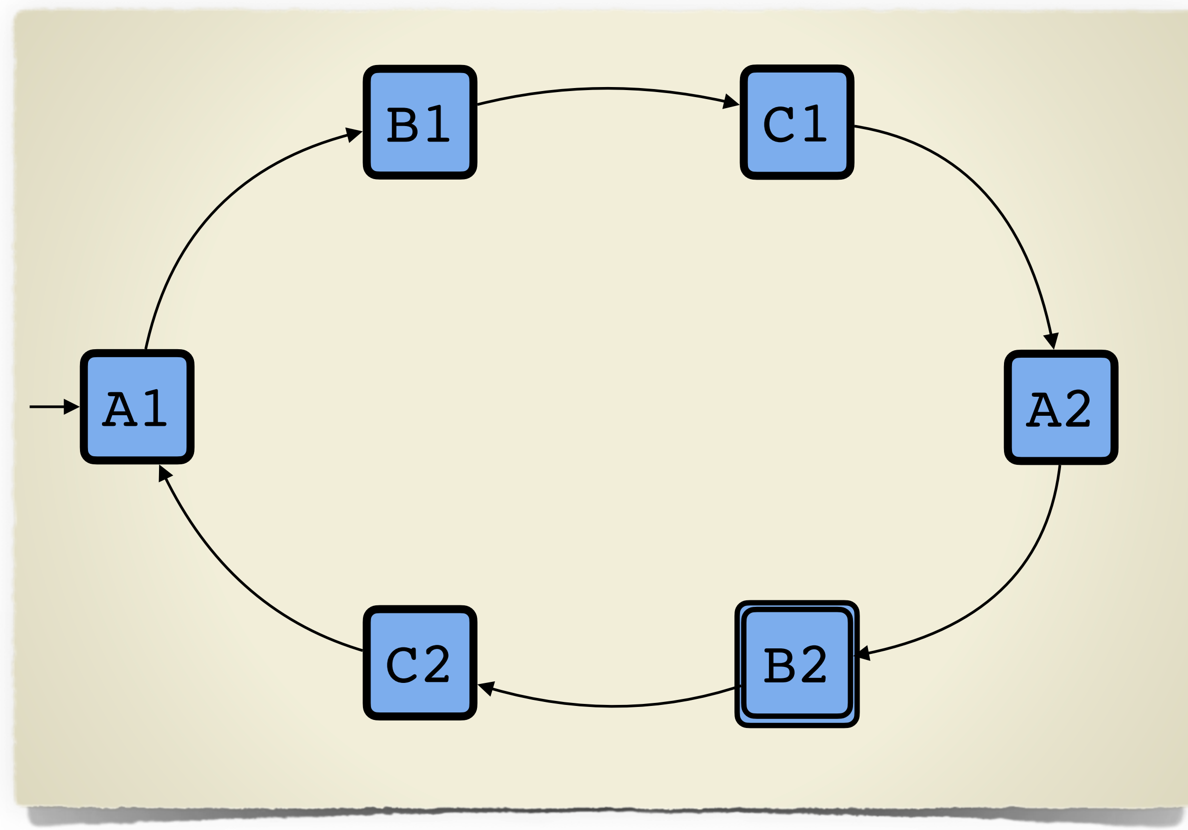

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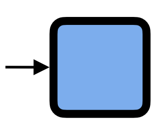
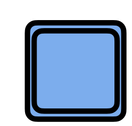
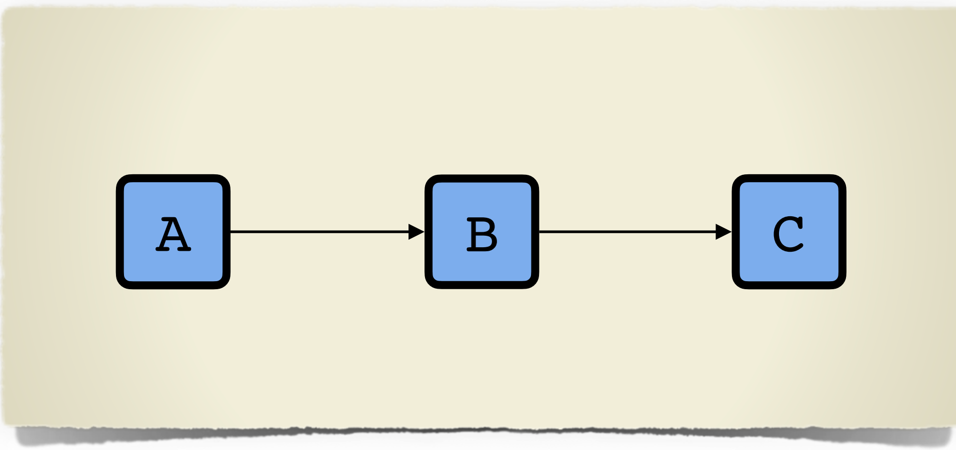
- Start nodes: 
- Target nodes: 

Path Representations: Examples

The path from A to C twice



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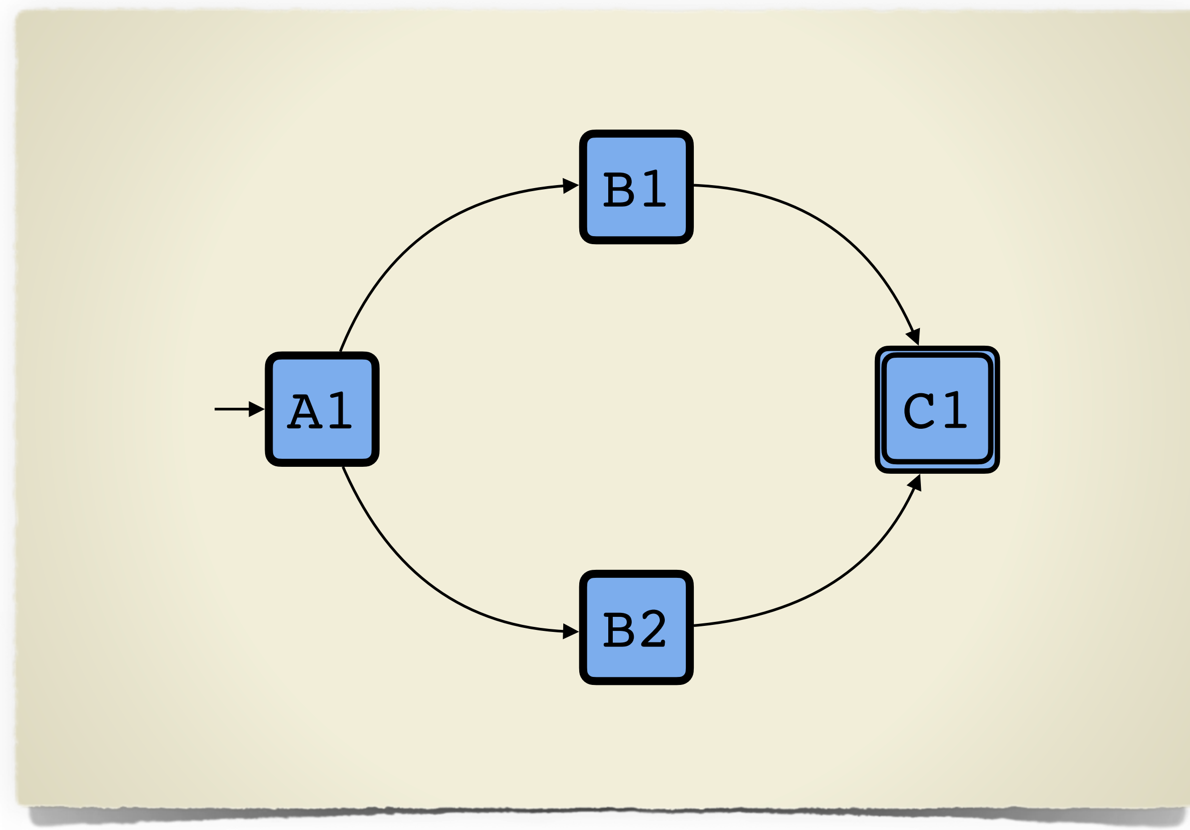

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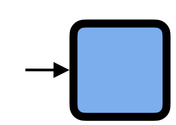
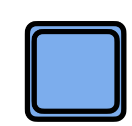
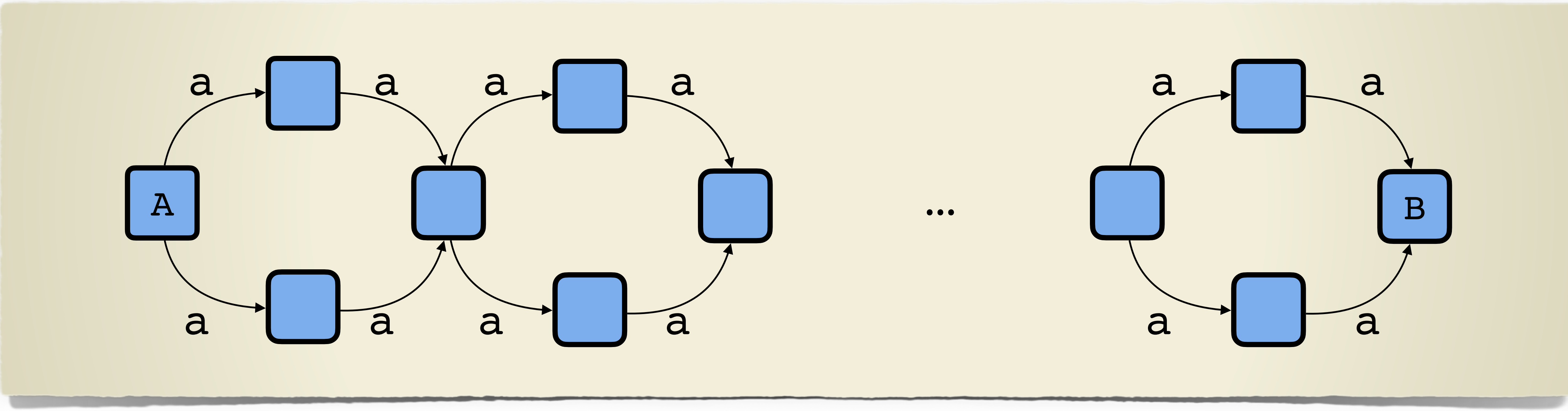
- Start nodes: 
- Target nodes: 

Path Representations: Examples

The 2^n paths from A to B



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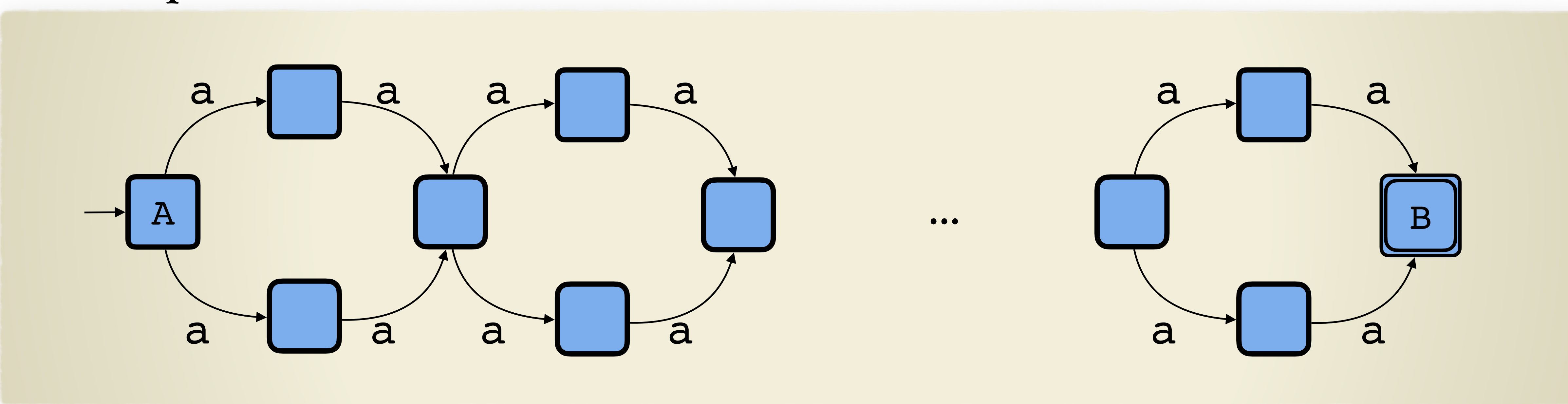
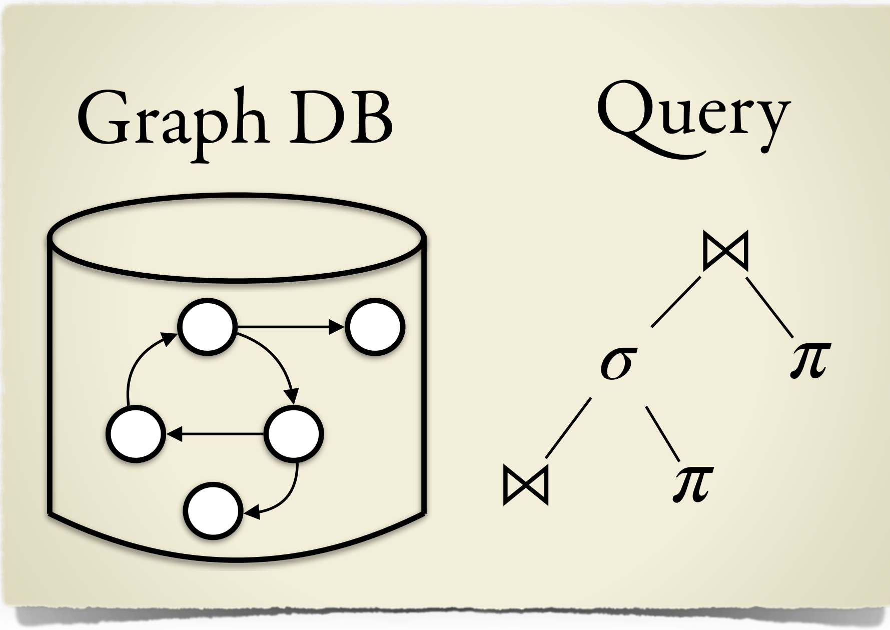


Path Representations: Envisioned Use



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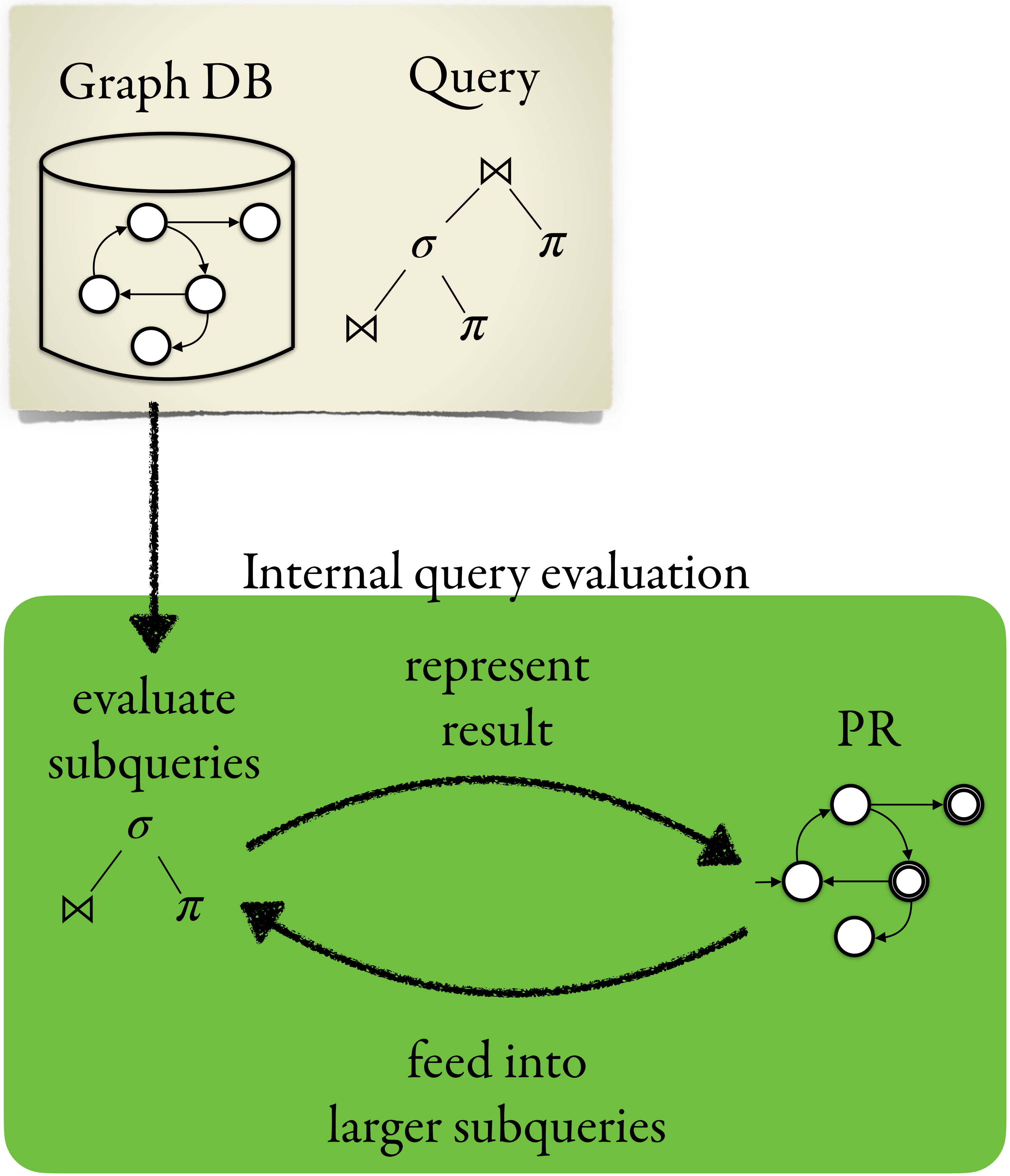

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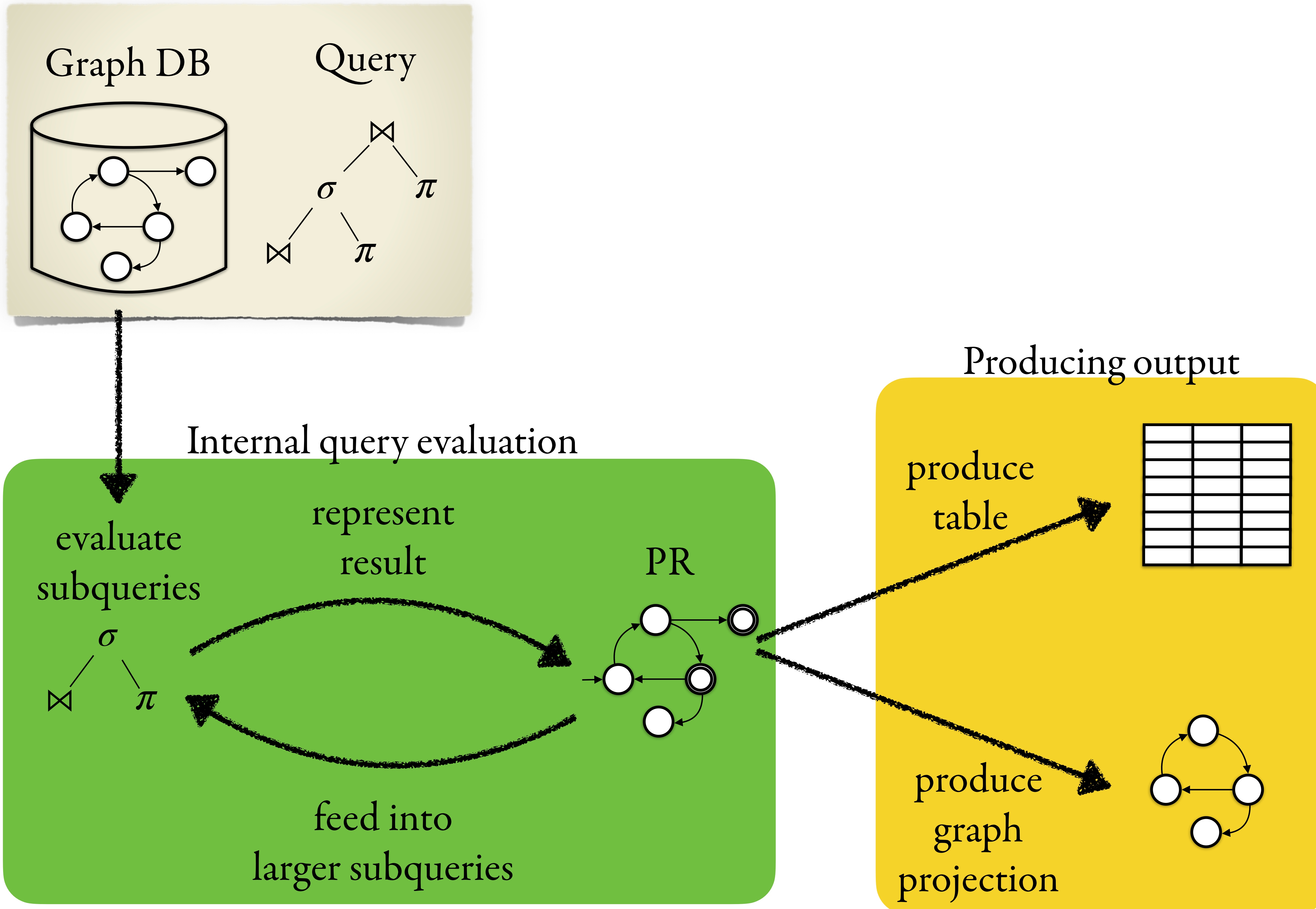

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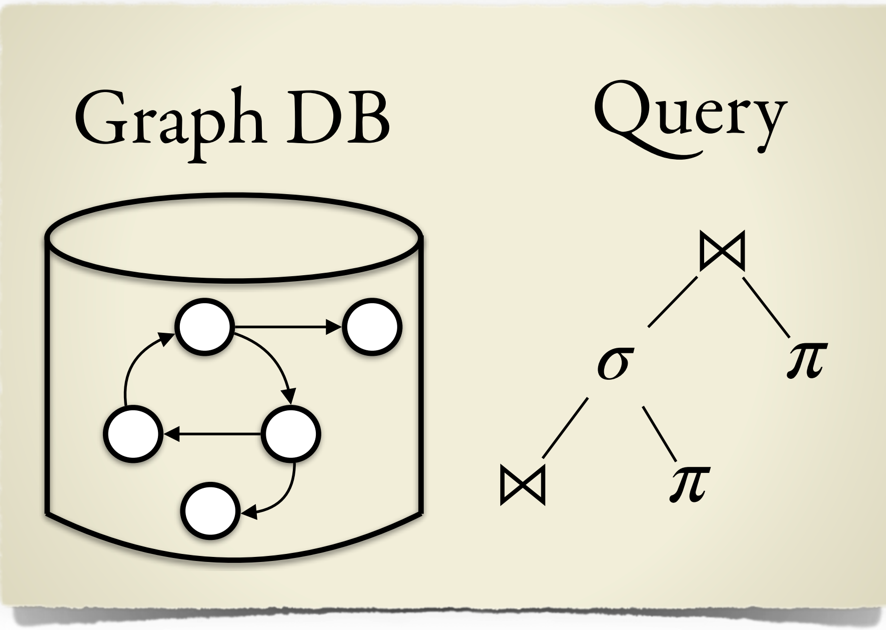

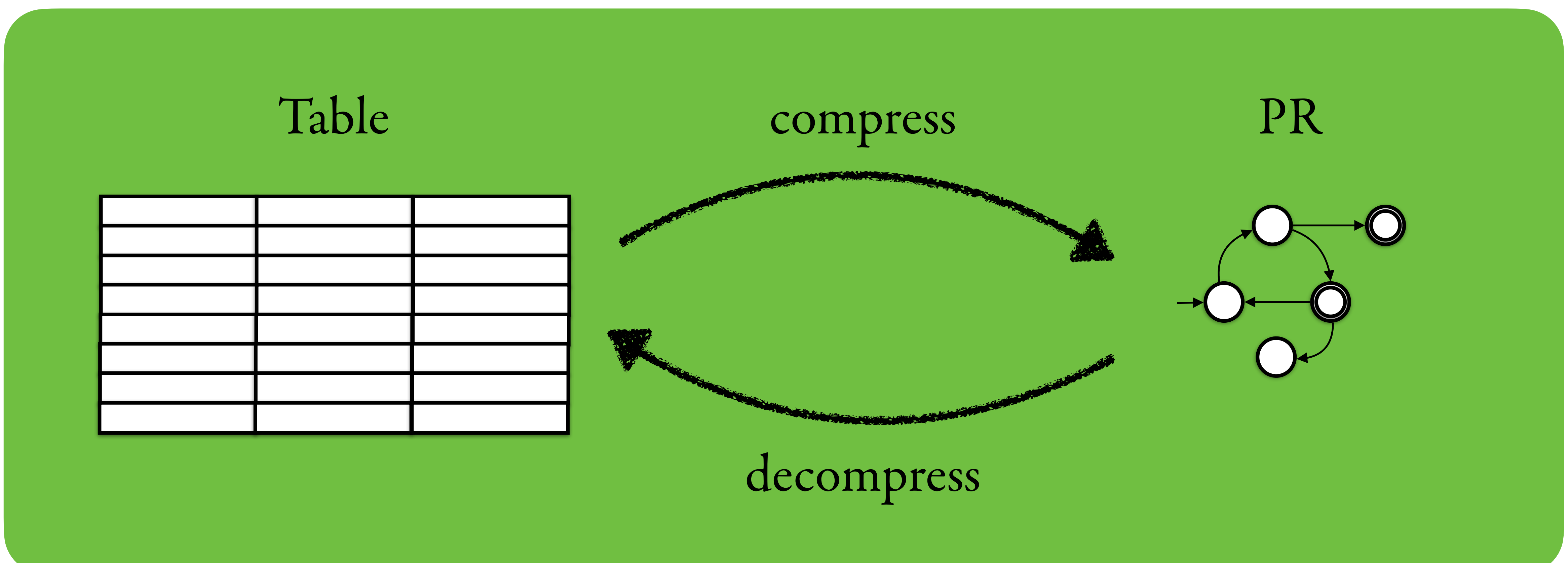

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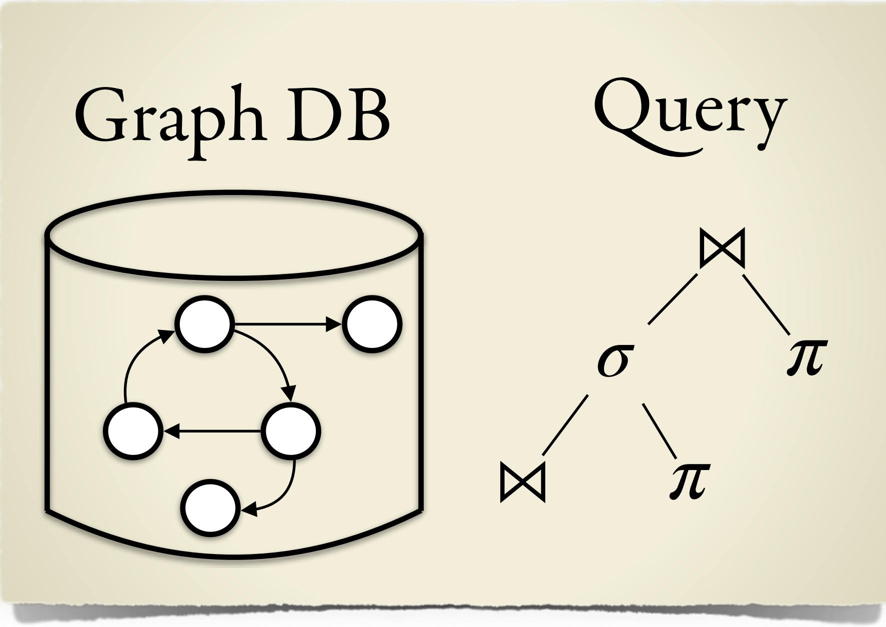
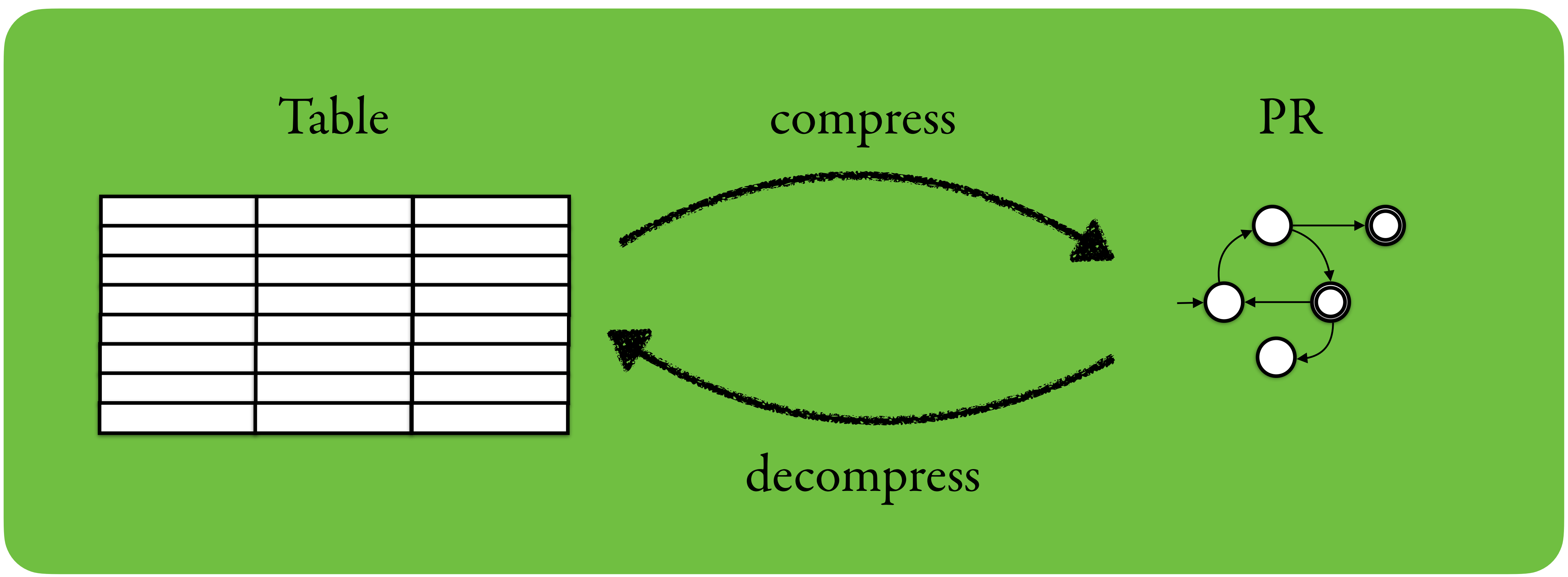


What we investigate(d)

Size of representation
Losslessness / Expressivity
Complexity of computing a PR
Complexity of applying upstream operators
Complexity of producing output



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- You can compute the table back from the PR
  - This costs about linear time in the size of the table

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Representing Multisets of Paths

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In our draft paper, we study PRs for RPQs under different evaluation modes:

- all paths
- all shortest paths
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shortest, lexicographically shortest ⇝ similar

simple paths, trails ⇝ more expensive,
but PRs are still exp more succinct than tables

PRs for Query Evaluation

Regular Path Queries

From such a PR, we can

- count the number of paths in polynomial time
- uniformly sample a path of length n in polynomial time

Beyond RPQs?

Unions of Regular Path Queries

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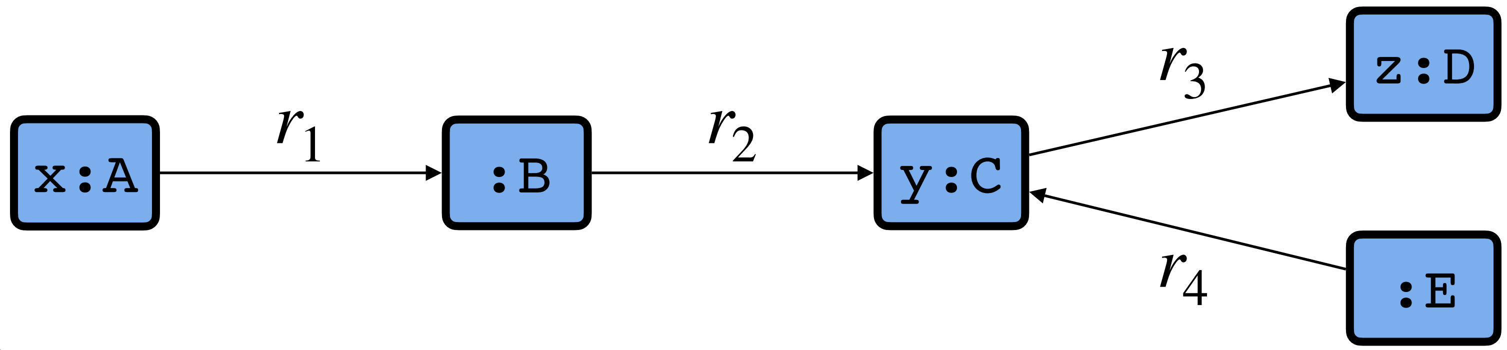
- PRs open up interesting aspects of query optimization

We're looking into those

Conjunctions of (2)RPQs

Take the query

$\pi_{x,z}$ (



)

Conjunctions of (2)RPQs

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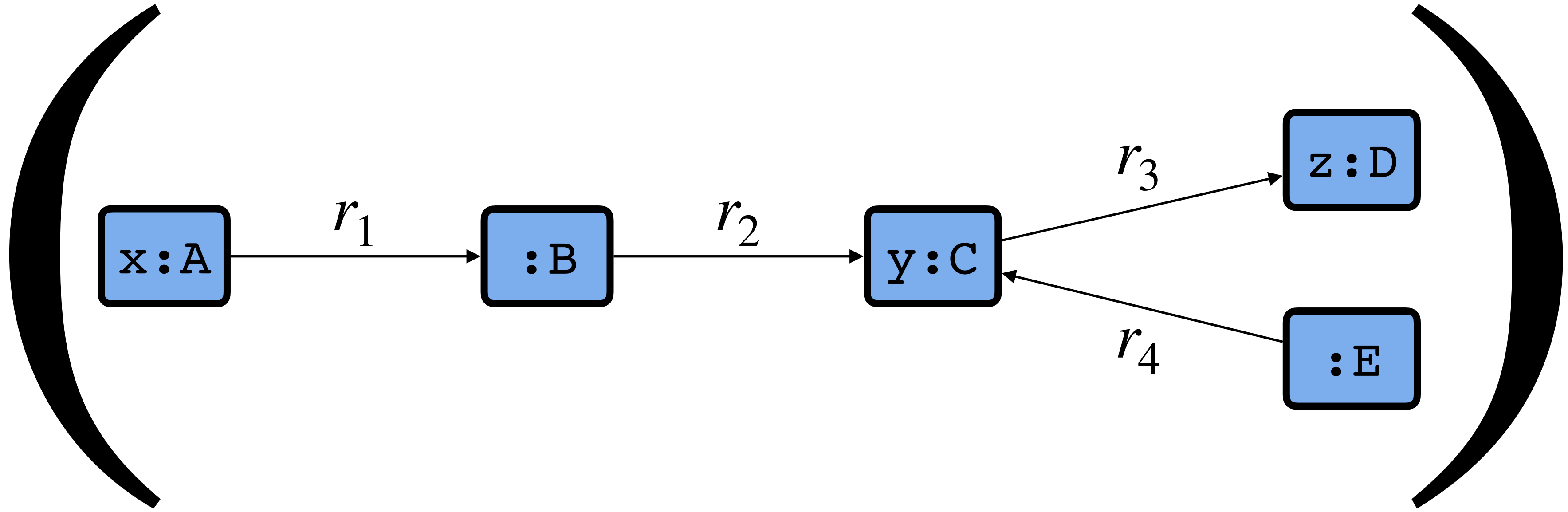

Lemma

For a given set of nodes U and an RPQ r, you can compute in linear time

- the set V such that there's a path
  - from some node in U
  - to some node in V
- a PR that contains all these paths

Step 1:
Take the A-nodes of the graph, apply the lemma to get candidates for :B

Conjunctions of (2)RPQs

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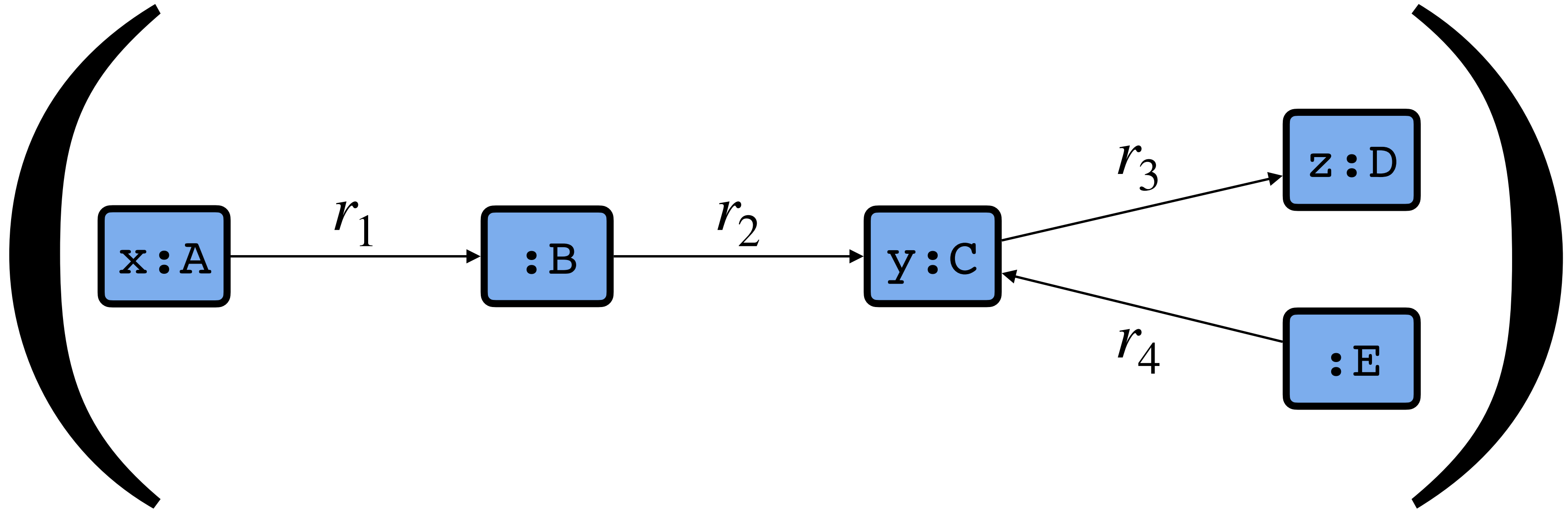

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(With tables for intermediate results, already this step costs exponential time)

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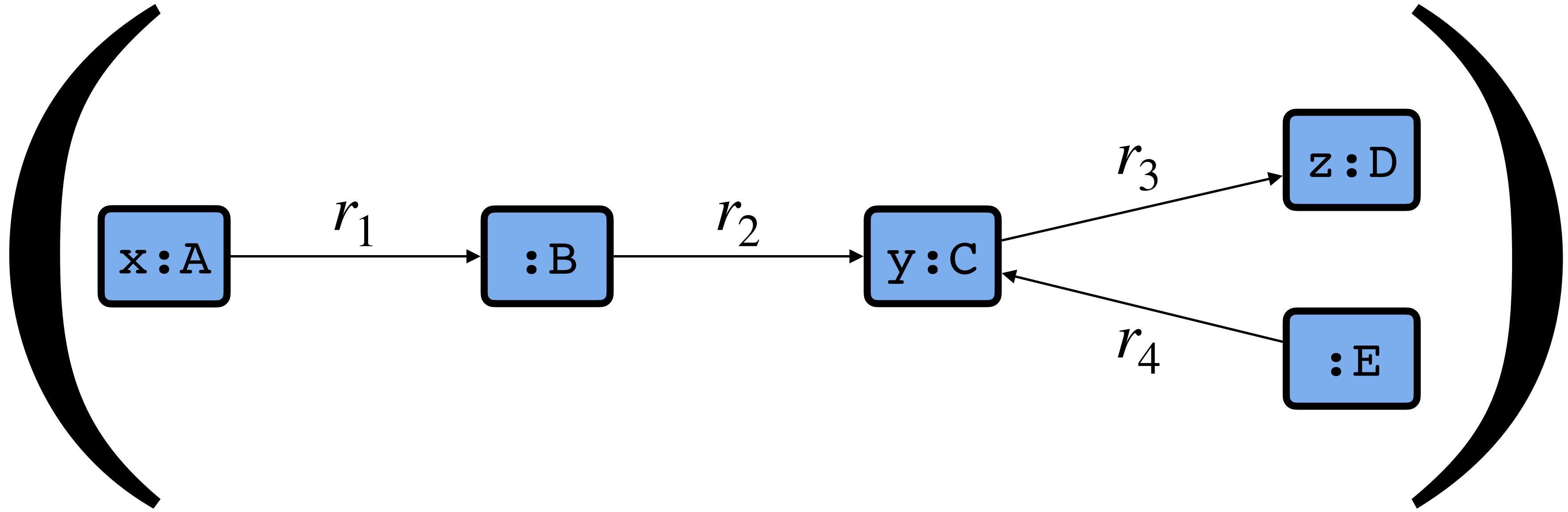
$\pi_{x,z}$ 

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Step 2:
Apply the lemma again to get candidates for z:D and :E

Step 3:
Trim everything; using backward reachability

Conjunctions of (2)RPQs

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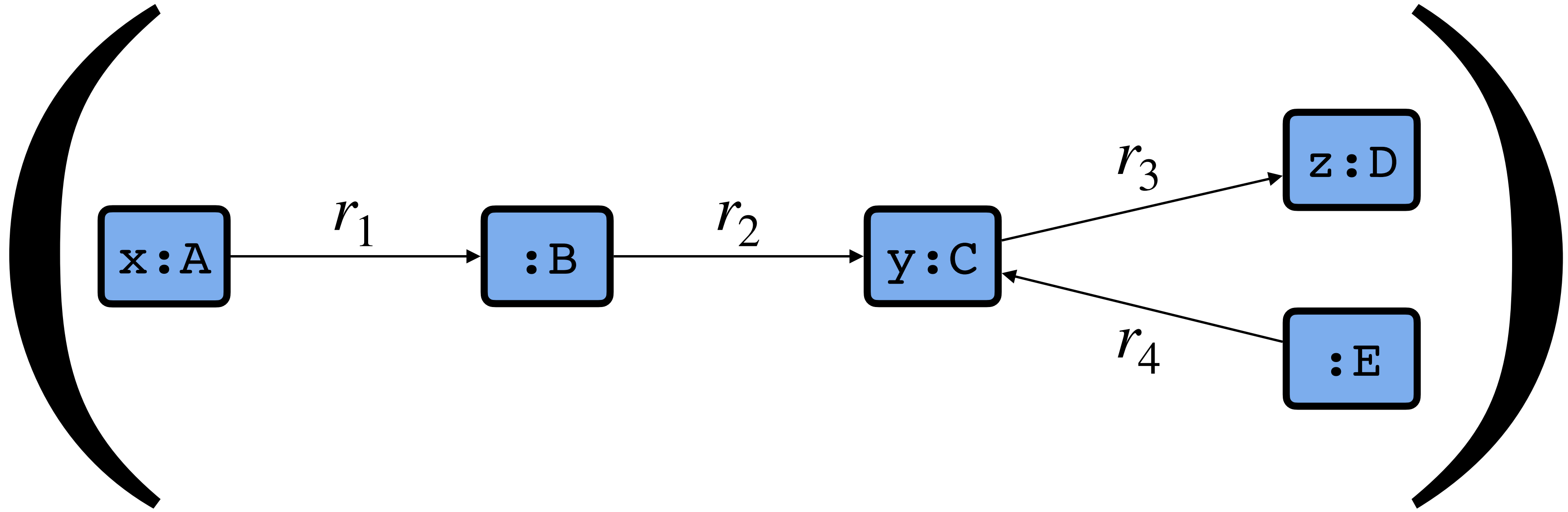
$\pi_{x,z}$



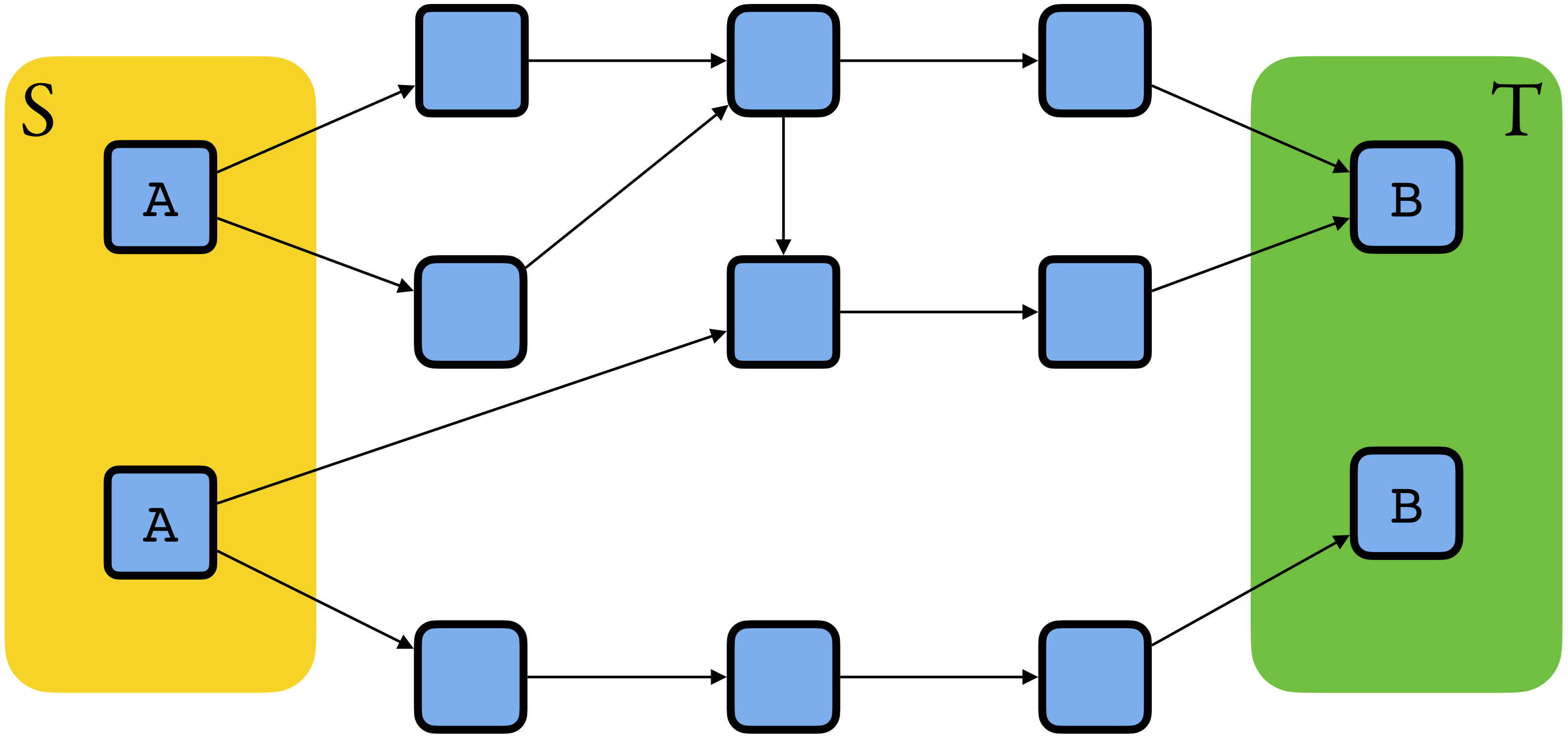
Lemma

For a given PR R of G and a pair of nodes (u, v) of G, we can compute the number of paths from u to v represented by R in linear time

Step 4:
Use counting results to efficiently count cardinalities of endpoint pairs in the result

Conjunctions of (2)RPQs

Insight

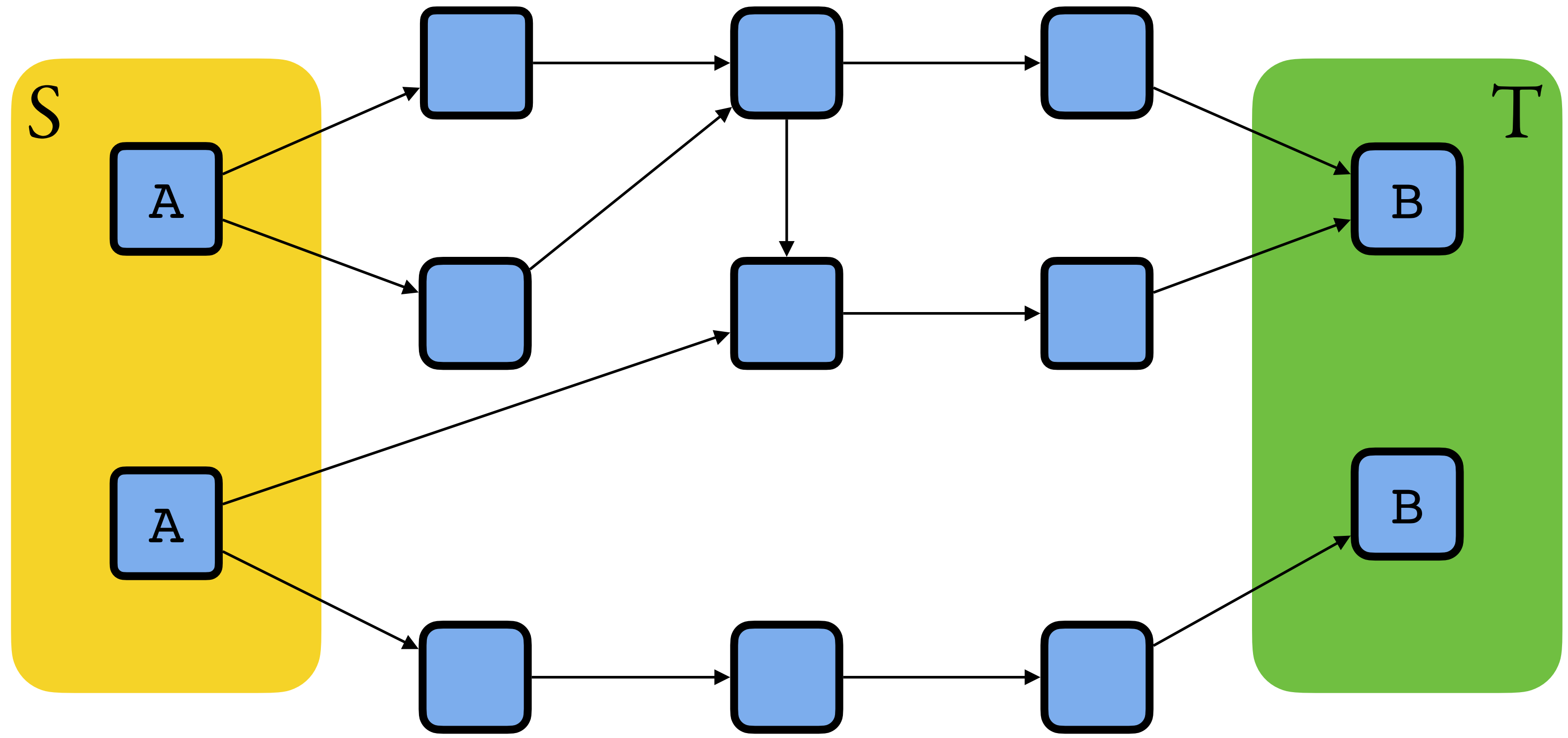


Using PRs, we can represent
"all paths from A-nodes to B-nodes"
in different ways

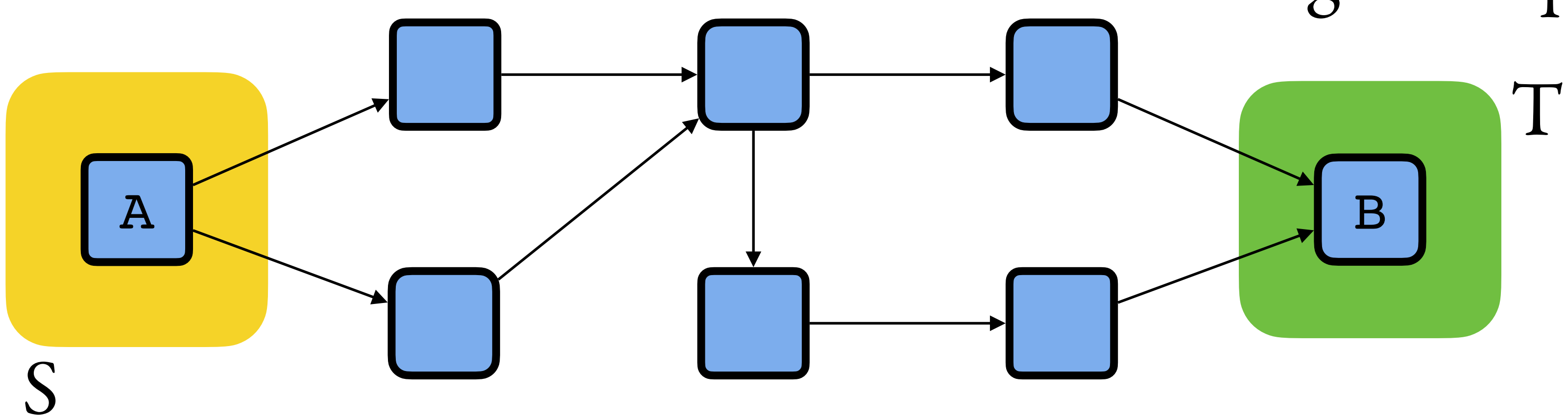
1. As you see it here

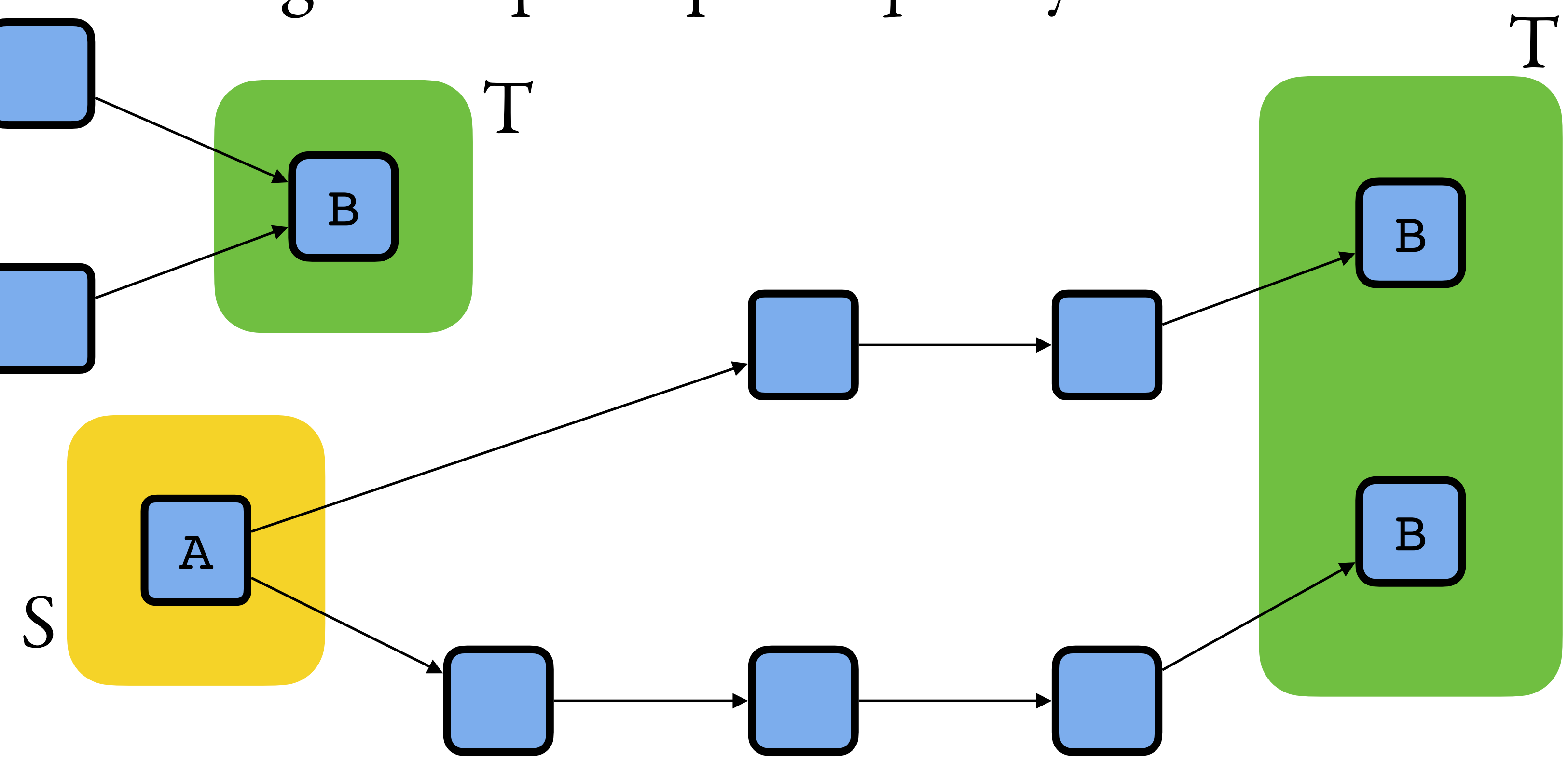
Conjunctions of (2)RPQs

Insight



2. In a way that allows you to get "endpoint pairs" quickly





This gives you a lot of flexibility for CRPQ evaluation

Concluding

1. The exponential output challenge
2. The composability challenge
3. The "output representation" challenge

1. PRs are succinct, so they may help a lot
2. PRs are graphs, so they may help here too
3. We're not HCI experts, so we don't know how PRs help users to digest results (but who knows?)

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Our contribution

We introduce the concept of PRs
that we believe can become quite helpful
for evaluating modern graph DB queries
in which paths are first-class citizens

Thanks!

Questions?

--> happy to chat here

--> feel free to reach out by email