Regularities in bisimulation reductions of big graphs

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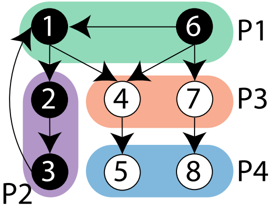
Bisimulation reduction of graphs

- Bisimulation partitioning is an important concept in many fields (computer science, modal logic, etc.), in DB research as well (structural index, graph reduction)
- It can be seen as a way of clustering nodes



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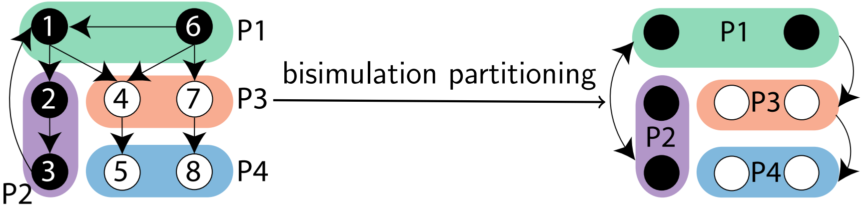

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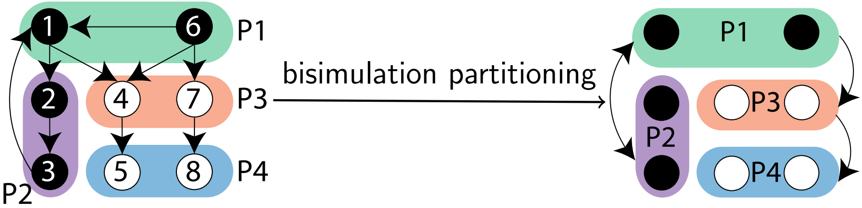

- Reduce graph size while preserving structural properties (e.g., reachability)
- Result can be seen as a (PB) graph
- What properties does the PB graph have?

Questions

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- Do results under bisimulation reduction (e.g., PB graph) also have such properties?
- How would that knowledge help us?

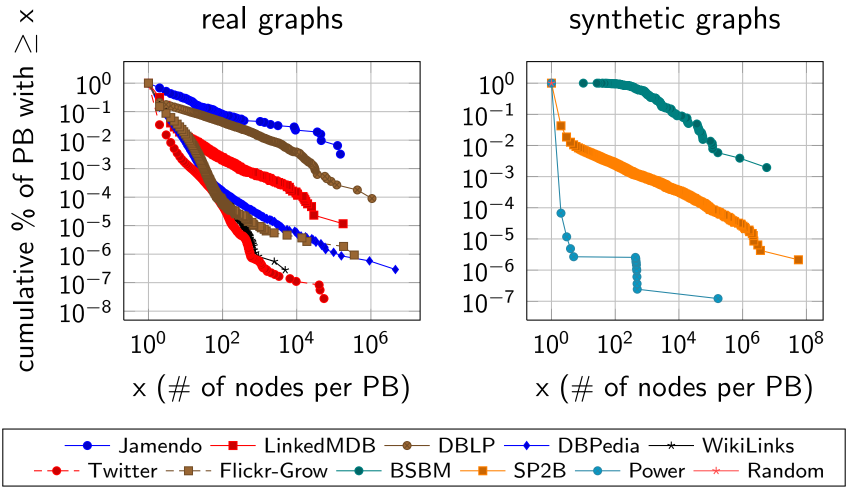
Experimental setup for investigation

- Big graphs, from 1 Million to 1.4 Billion edges (Twitter, DBPedia, etc.)
- State-of-the-art external-memory algorithm for computing bisimulation reductions
- We use cumulative distribution function (CDF) to present distributions

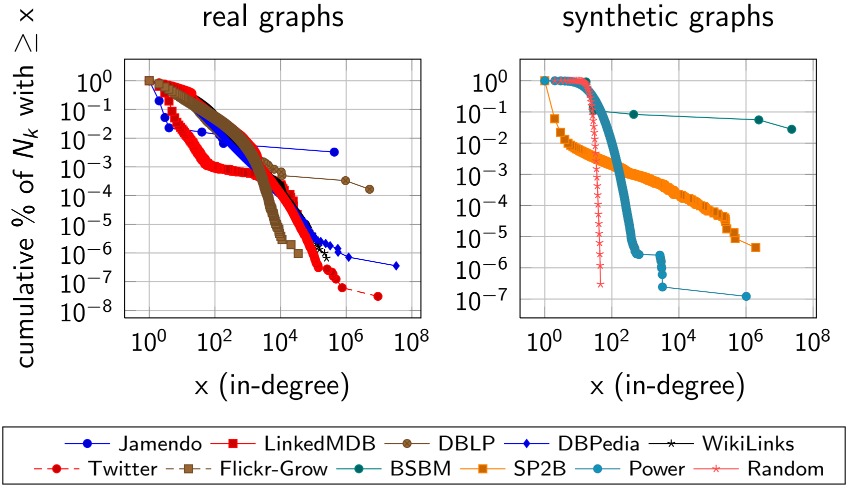
Regularities - bisimulation result

Power-law also exists in many attributes for bisimulation partition results for *real graphs*. But this is not the case for *synthetic graphs*.

Regularities - partition block size distribution



Regularities - PB graph in-degree distribution



A close look at Social Intelligence Benchmark (old)

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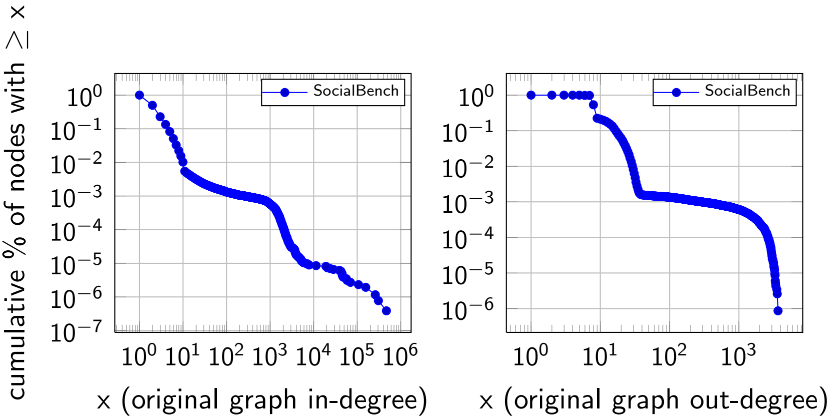
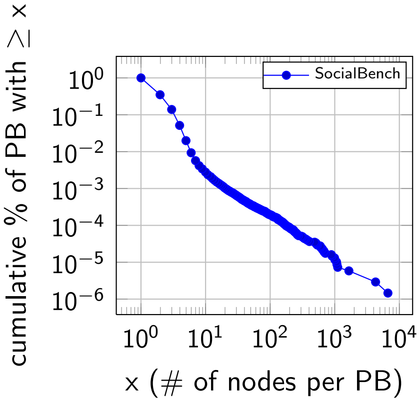
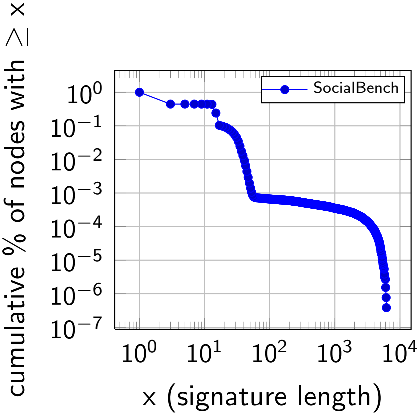
- What structure is exhibited by graphs generated by the Social Intelligence Benchmark?
- Use s3g2130313.tar, downloaded from `sourceforge.net/projects/sibenchmark/` (thanks to Minh-Duc Pham)
- Number of nodes: 2.6M, Number of edges: 12.6M
- Configuration: numtotalUser: 10000, 2010-1-1 to 2012-1-1

In-degree and out-degree of original graph

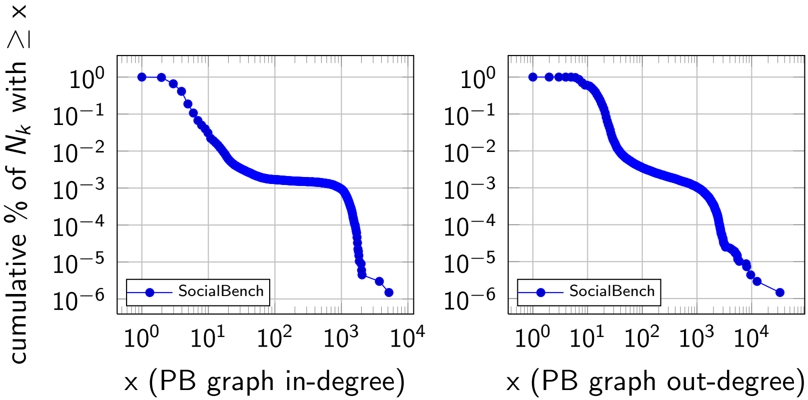


Partition block size & signature length distribution





In-degree and out-degree of PB graph



Insights

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- Power-law distributions in bisimulation results emphasize the fact that data skews are expected in applications (indexes, data partitioned among machines, ...)
- Some more work remains to be done for synthetic graph generators towards exhibiting the reduction properties of real graphs.
- Bisimulation result/graph grows when original graph grows, which calls for scalable/adaptive algorithms (e.g., choose different k for different parts of the graph, different node/edge labeling)

Thank you! Q&A

For more information, just google **seeqr project** or

visit: `bit.ly/seeqr`

Definition of *k-bisimilar*

Definition

Let k be a non-negative integer and $G = \langle N, E, \lambda_N, \lambda_E \rangle$ be a graph. Nodes $u, v \in N$ are called *k-bisimilar* (denoted as $u \approx^k v$), iff the following holds:

1. $\lambda_N(u) = \lambda_N(v)$,
2. if $k > 0$, then for any edge $(u, u') \in E$, there exists an edge $(v, v') \in E$, such that $u' \approx^{k-1} v'$ and $\lambda_E(u, u') = \lambda_E(v, v')$, and
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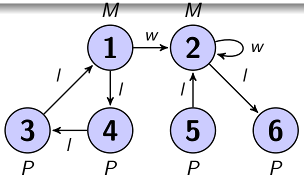
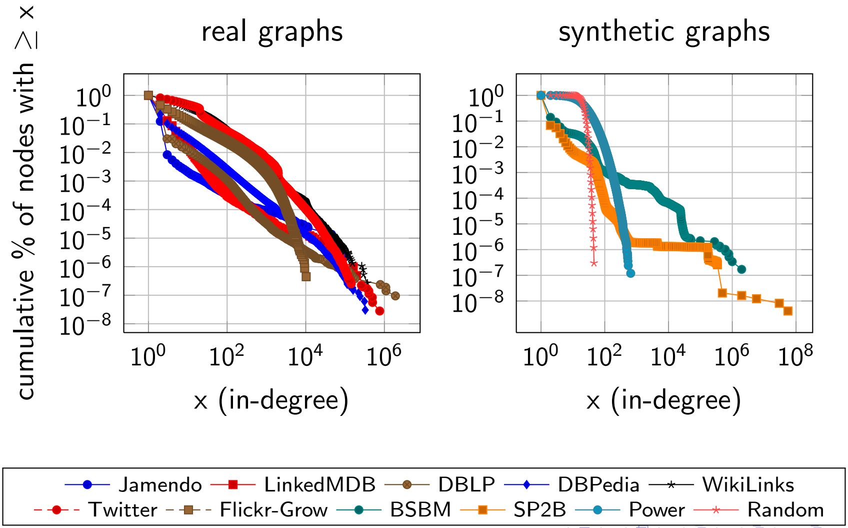


In this example graph, nodes 1 and 2 are 0- and 1- bisimilar but not 2-bisimilar.

In-degree distribution of original graphs



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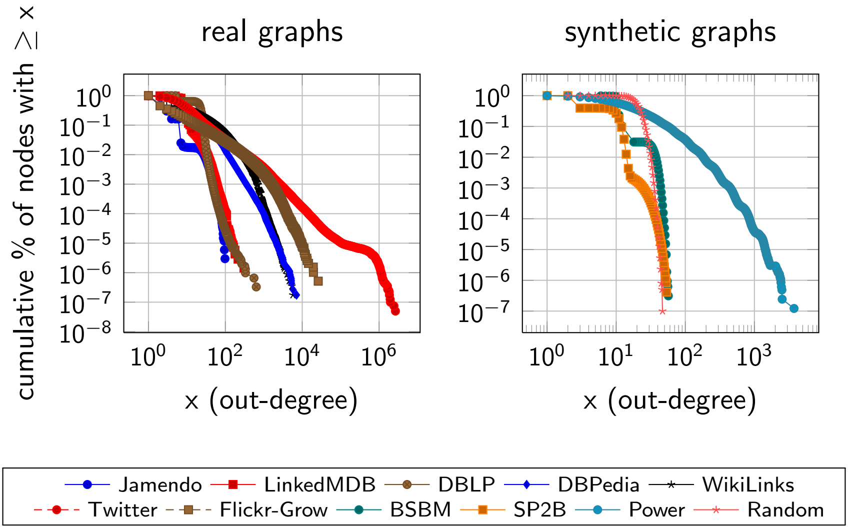
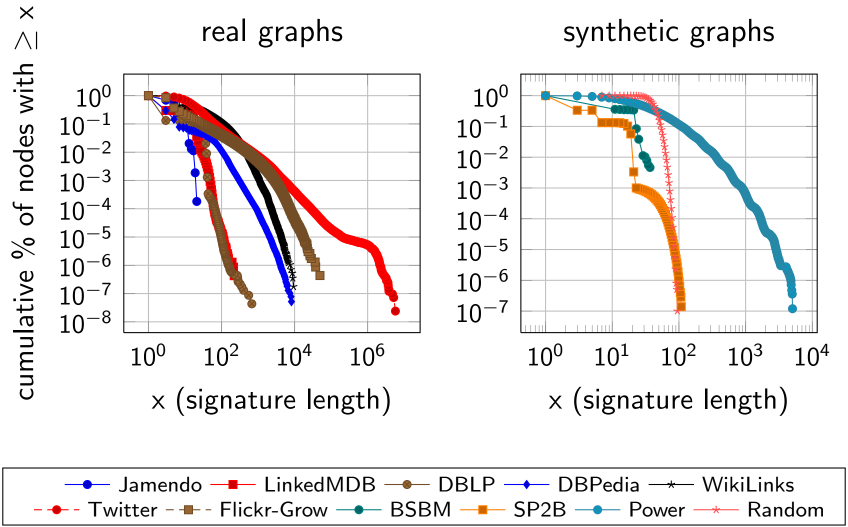
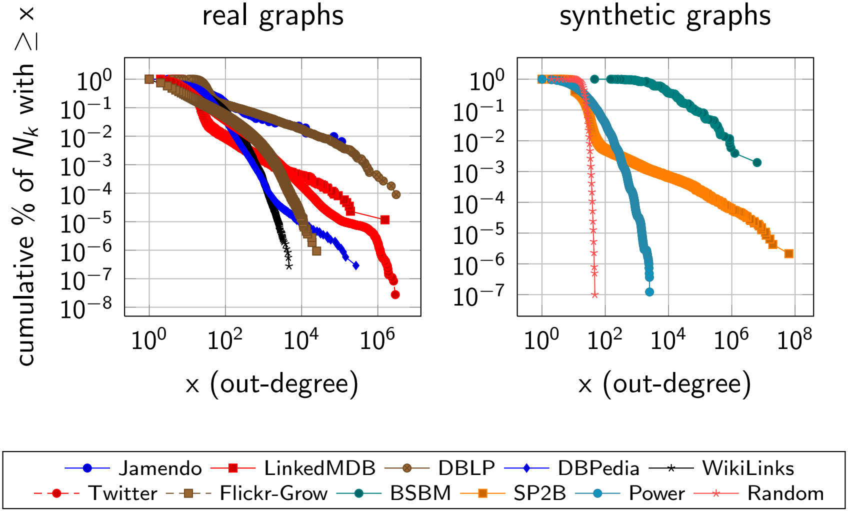


Signature length



Out-degree of PB graph



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  - Yes.
- How fast does it grow?
  - Linearly with respect to the original graph.